Fuzzy Control of One Gear Over Other

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1. Introduction

A classic example of an unstable control system is that of a small ball is placed on top of a fixed large ball.

The small ball can never hold its position and will roll off the large ball immediately due to the force of the gravity. However, even for such un unstable system, the small ball can be made to stay on top of the large ball by properly controlling the large ball's rotation.

This report proposed that given two gears, one placed on top of the other and the lower gear connected to a DC motor, the top gear can be made to stay in place by applying fuzzy control to the DC motor. The experimental gear set is now under construction. Simulation has been done to find the stable condition by using numerical data of the parameters in the set.

2. Gear set

Principles of the gear set is shown in Fig. 1, and Fig. 2 depicts the experimental system. Two gears are mounted one on top of the other. The gears are engaged and their centers are fixed by a connecting lever to prevent separation. A DC motor is connected to the lower gear through a gear box. The inclination angle a of the upper gear is measured by a potentiometer connected to the connecting lever through a sensing lever.

A **2048**〔p/rev〕 Pulse encorder is mounted on the DC motor's shaft, and measures the DC motor angle's θ.

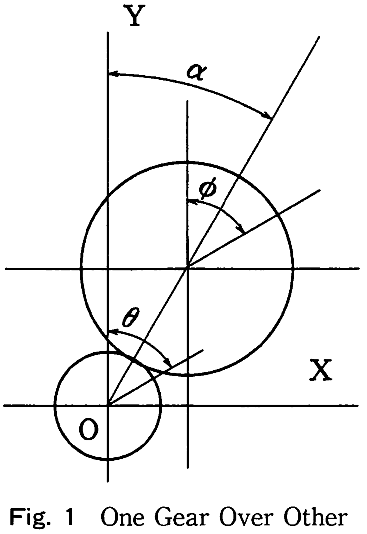


Fig. 1 One Gear Over Other

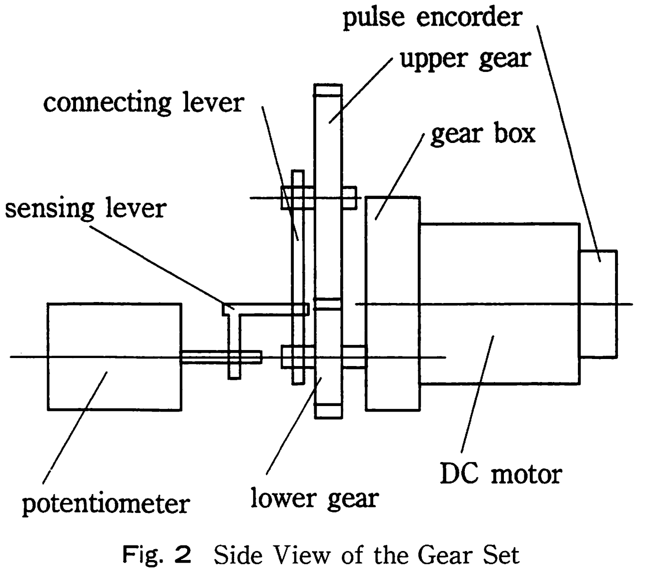


Fig. 2 Side View of the Gear Set

3. Simulation model

Table 1 lists the parameters and their numerical data of the experimental gear set.

Table 1 Data of the Experimental Gear Set

Symbols	Quantity	Numerical Data	Units
	Lower gear		
J_1	Equivalent moment of inertia	8.52×10^{-6}	$[Kg\cdot m^2]$
r_1	Radius	0.04	$[m]$
	Upper gear		
m_2	Weight	6.63×10^{-2}	$[Kg]$
J_2	Moment of inertia	1.65×10^{-5}	$[Kg\cdot m^2]$
r_2	Radius	2.5×10^{-2}	$[m]$
l	Center distance of both gear	4×10^{-2}	$[m]$
D_θ	Equivalent friction coefficient of lower gear	4.68×10^{-5}	$[N\cdot m/rad/s]$
K_θ	Voltage coefficient of lower gear angle	5.38×10^{-2}	$[V/rad/sec]$
K_a	Voltage coefficient of lever inclination angle	6.36	$[V/rad]$
K_v	Voltage coefficient of motor torque	7.8×10^{-3}	$[Nm/V]$

The state space equation is calculated by using the parameters in Table 1. The upper gear is engaged with the lower gear, the rotation angle of the upper gear is $\Phi = \frac{r_1}{r_2}(a-\theta)$

The coordinates of the center of the upper gear is $(l \sin a, l \cos a)$

Kinetic energy is $T = \frac{1}{2}J_1\dot{\theta}+\frac{1}{2}m_2l^2\dot{a}^2+\frac{1}{2}J_2\left\{\frac{r_1}{r_2}(\dot{a}-\dot{\theta})\right\}^2$

Potential energy is $U = m_2gl \cos a$

Dissipation energy is $D = \frac{1}{2}D_\theta\dot{\theta}^2$

Generalized coordinates q_i are a, θ, and generalized force Q_i is f_θ. And the above energy values are substitute into the Lagrange's equation which is shown bellow.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial T}{\partial \dot{q}_i}+\frac{\partial U}{\partial q_i} = Q_i$$

If the upper gear is stabilized, then a is kept small value. This justified the approximation $\sin a \fallingdotseq a$.

The lower gear equivalent moment of inertia of the upper gear J_3 is $J_3 = J_2\left(\frac{r_1}{r_2}\right)^2$

The differential equation for the gear set can then be written as shown below.

$$\ddot{a} = \frac{(J_1+J_3)\,m_2gl}{(J_1+J_3)\,(m_2l^2+J_3)-J_3^2}a-\frac{J_3D_\theta}{(J_1+J_3)\,(m_2l^2+J_3)-J_3^2}\dot{\theta}+\frac{J_3}{(J_1+J_3)\,(m_2l^2+J_3)-J_3^2}f_\theta$$

$$\ddot{\theta} = \frac{m_2glJ_3}{(J_1+J_3)\,(m_2l^2+J_3)-J_3^2}a-\frac{(m_2l^2+J_3)\,D_\theta}{(J_1+J_3)\,(m_2l^2+J_3)-J_3^2}\dot{\theta}+\frac{m_2l^2+J_3}{(J_1+J_3)\,(m_2l^2+J_3)-J_3^2}f_\theta$$

The state variables are chosen to be

$$\mathbf{x} = [\theta \quad a \quad \dot{\theta} \quad \dot{a}]^T \qquad f_\theta = u$$

And the state space equation and output equation are as follows.

$$\mathbf{x} = \mathbf{A}\mathbf{x}+\mathbf{b}u$$

$$y = \mathbf{c}\mathbf{x}.$$

where

$$\mathbf{A}=\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{m_2 g l J_3}{(J_1+J_3)(m_2 l^2+J_3)-J_3^2} & \dfrac{-(m_2 l^2+J_3)D_\theta}{(J_1+J_3)(m_2 l^2+J_3)-J_3^2} & 0 \\ 0 & \dfrac{(J_1+J_3)m_2 g l}{(J_1+J_3)(m_3 l^2+J_3)-J_3^2} & \dfrac{-J_3 D_\theta}{(J_1+J_3)(m_2 l^2+J_3)-J_3^2} & 0 \end{pmatrix} \qquad \mathbf{b}=\begin{pmatrix} 0 \\ 0 \\ \dfrac{m_2 l^2+J_3}{(J_1+J_3)(m_2 l^2+J_3)-J_3^2} \\ \dfrac{J_3}{(J_1+J_3)(m_2 l^2+J_3)-J_3^2} \end{pmatrix}$$

$$\mathbf{c}=[0 \quad 1 \quad 1 \quad 0]$$

Numerical data are substituted for each parameter in Table 1.

$$\mathbf{A}=\begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 9.76\times10 & -3.31 & 0 \\ 0 & 4.11\times10^2 & -1.75\times10^{-1} & 0 \end{pmatrix} \qquad \mathbf{b}=\begin{pmatrix} 0 \\ 0 \\ 7.43\times10^4 \\ 3.75\times10^3 \end{pmatrix}$$

Now, matrix **A**, **b** are transformed to a new equivalent matrix by similarity transformation.

$$\mathbf{x}^*=\mathbf{T}\mathbf{x} \qquad \mathbf{T}=\begin{pmatrix} K_\theta & 0 & 0 & 0 \\ 0 & K_a & 0 & 0 \\ 0 & 0 & K_\theta & 0 \\ 0 & 0 & 0 & K_a \end{pmatrix}$$

K_θ, K_a, and K_v are the voltage coefficients of the lower gear angle, the lever inclination angle and the motor torque respectively, and their numerical data are shown in Table 1. All state variables are expressed by voltage.

And the final state space equations are as follows.

$$\dot{\mathbf{x}}^*=\mathbf{A}^*\mathbf{x}^*+\mathbf{b}_v^* u$$

$$\mathbf{A}^*=\mathbf{T}\mathbf{A}\mathbf{T}^{-1}, \qquad \mathbf{b}_v^*=\mathbf{K}_v\mathbf{T}\mathbf{b}$$

$$\mathbf{A}^*=\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 8.24\times10^{-1} & -3.29\times10^{-1} & 0 \\ 0 & 4.08\times10^2 & -2.0\times10 & 0 \end{pmatrix} \qquad \mathbf{b}_v^*=\begin{pmatrix} 0 \\ 0 \\ 3.11\times10 \\ 1.85\times10^2 \end{pmatrix}$$

4. Simulation System

Fig. 3 shows the block diagram of the simulation system. Fuzzy reasoning is executed using the fuzzy simulation software FS-2000 developed by OMRON. The output signals of the gear set are the lever's inclination angle, its angular velocity and the angular velocity of the DC motor. These signals are named *VAA*, *VAV*, and *VTV* respectively. The input signal for the gear set is the motor driver's input voltage named *VIN*.

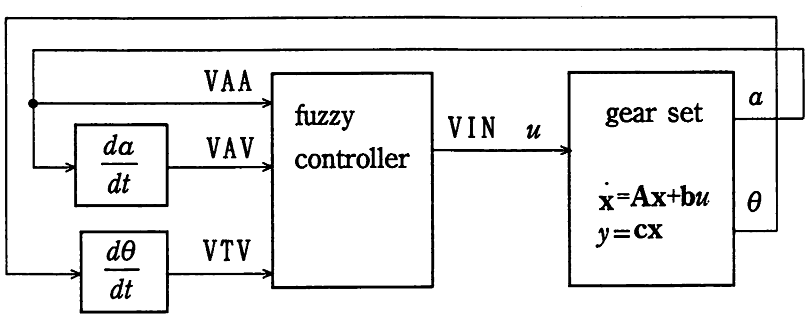
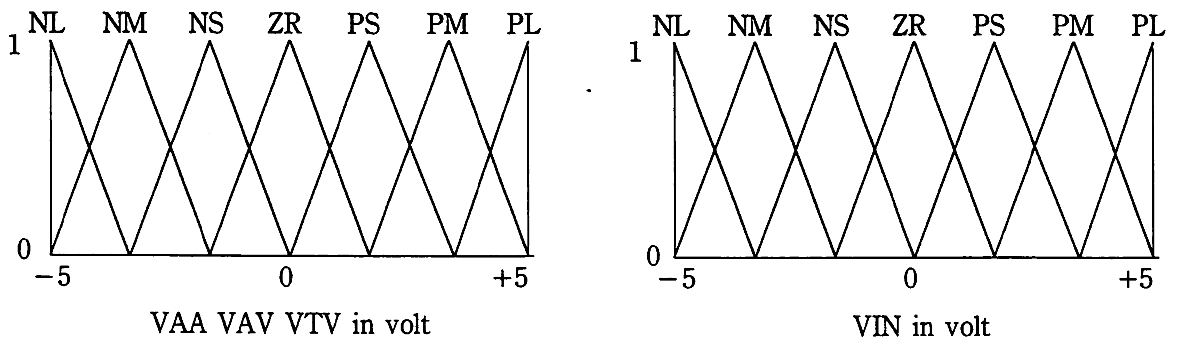


Fig. 3 Block Diagram of the Simulation System

5. Fuzzy control

5.1 Membership functions

In fuzzy reasoning section, the pre-condition variables are *VAA*, *VAV*, and *VTV*. Their membership functions are shown in Fig. 4(a). As for the conclusion variable *VIN*, its membership function is shown in Fig. 4(b). *PL*, *PM* etc. are fuzzy labels. There are 7 standard labels in both the membership functions.



(a) Pre-condition (b) Conclusion

Fig. 4 Membership Functions

5.2 Fuzzy rules

Table 2 is the fuzzy rule table. Lever angle and its velocity are used so as to make a is zero, which means that the upper gear is staying on top of the lower gear.

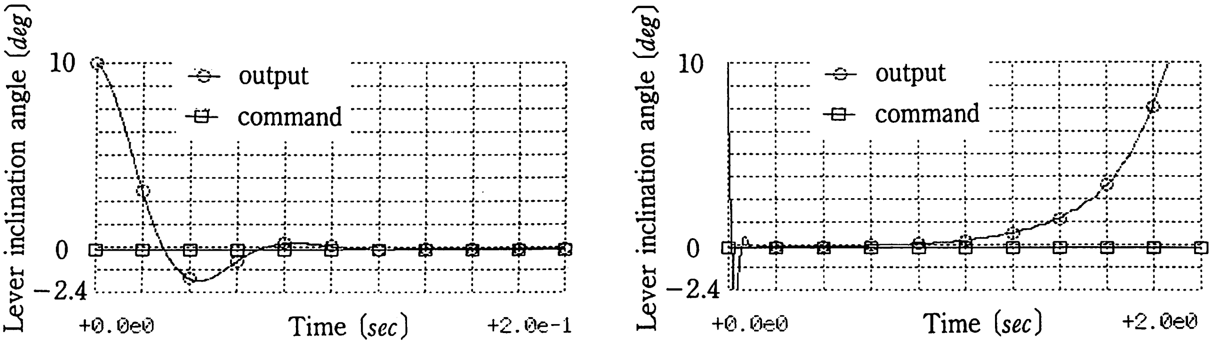
Table 2 Fuzzy Rule Table

		VAA : Lever angle						
		NL	NM	NS	ZR	PS	PM	PL
	NL				NL			
	NM				NM			
VAV :	NS				NS			
Lever angle	ZR	NL	NM	NS	ZR	PS	PM	PL
velocity	PS				PS			
	PM				PM			
	PL				PL			

6. Simulation results

Fig. 5(a) shows that the upper gear is able to reach to its top position on top of the lower gear within 30 ms.

Fig. 5(b) is the reduced time scale of Fig. 5(a). In this graph, it shows that after the upper gear reached its top position, it gradually rolled down after 2 seconds. The reason seems to be that the drive power of the lower gear is the same in each direction, so the velocity of the lower gear speeds up in one direction.



(a) Short time response

(b) Reduced time scale

Fig. 5 Simulation Result

7. Conclusion

The upper gear can be made to reach its top position and stay there by fuzzy control of the lower gear. The time for it to reach its top position is about 30 ms. But for now, it rolled back down after that. To increase the time the upper gear can stay at its top position, improvements to the fuzzy rule so that different power can be applied to the lower gear when it rotates in different directions, and the control of the rotation angle of the lower gear are being considered.

References

1) K.R. Symon : Mechanics, Addison-Wesley Publishing Company.
2) K. Tanoshima : Fuzzy Control of a Two-Dimensional Inverted Pendulum, Research Reports of Akita National College of Technology, 30 (1994), 67-70.