Fuzzy Control of a Two-Dimensional Inverted Pendulum

Katsuhide TANOSHIMA

(1994年 8 月22日受理)

1. Introduction

An inverted pendulum is often used as the research subject of control problems in an unstable multi-variable system[1]. It has been reported that good control of the pendulum can be achieved in systems using fuzzy control methods[2,3]. These systems are made up of a cart that moves along a belt driven by DC motor. A pendulum about an axis is fixed to the cart. The pendulum can be balanced vertically by controlling the DC motor.

This report proposes that the pendulum be made to be able to fall in not just one, but in all directions. Two wheels, which speed and rotating direction are controlled independently, are attached to the cart to allow it to move freely on the floor. In this manner, the pendulum on the cart can balance vertically in a two-dimensional space.

2. System composition

2.1 Cart

The composition of the cart is depicted in Fig. 1 and its specifications are shown in Table 1. Fig. 2 is a photograph of the cart. A wheel, connected to the DC motor through a gear box, is arranged on each side of the cart. Both wheels can rotate independently. They will run at the same speed and in the same direction when the cart is moving forward or backward. However, to change the cart's direction of movement, the speed and turning direction of each wheel may differ. The left and right wheel can be driven separately and the driving signals to them are named LDR and RDR respectively. Besides the wheels, two roller casters are attached to the front and the rear of the cart to privent it from falling. At the center of the cart, a joystick is mounted with its arm fixed to the lower end of the pendulum. The joystick can detect the inclination angle of the pendulum in both X and Y directions independently by the change in the resistance value. The resistance changes are converted to voltage and the signals are named XPS and YPS.

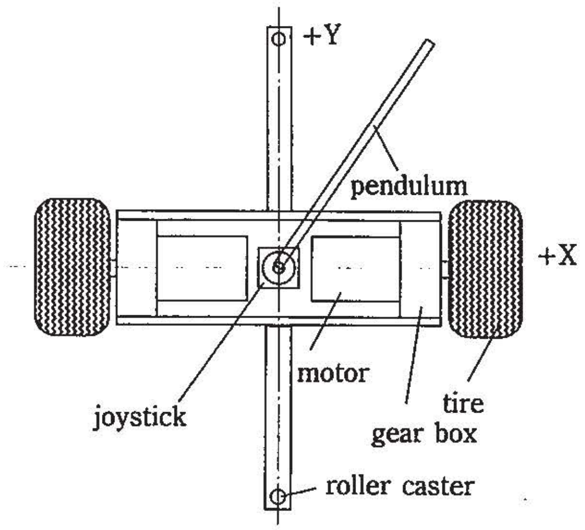

Fig. 1 Composition of the Cart

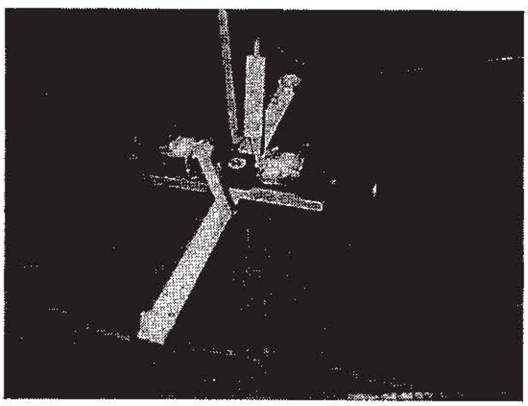

Fig. 2 Photograph of the Cart

Table 1 Specification of the Cart

cart	net weight	2.1 Kg
	CD of wheels	320 mm
	dia. of tire	85 mm
pendulum	length	1 m
motor	voltage	24 V
	current	0.8 A
	moment of inertia	8.2X10^{-6} Kgm^2
	torque constant	0.039 Nm/A
gear box ratio		15:1 step down

2.2 Controller

The controller's block diagram is depicted in Fig. 3. Signal XPS and YPS from the joystiok are directly connected to an A/D converter. Their differentiated values (signal named XVL and YVL) are likewise connected to the A/D converter: After that, the four digitalized signals will go into the fuzzy reasoning board.

Finally, the resultant signals from the fuzzy reasoning board are connected to the DC motor driver through a D/A converter.

The fuzzy reasoning board FB-3098 contains the digital fuzzy processor FP-3000 developed by OMRON. This board and the A/D, D/A converter board are installed into the extension slots of a PC-98 personal computer. C language is used to control the system. The fuzzy rules and membership function are developed using the OMRON software fuzzy minicon FS-1000, and are transferred to the RAM in the FB-3098. During real time operation, rasonng is done by the instructions in this RAM. As a result, very high speed rasoning is achieved. Sampling timing is about 3 ms.

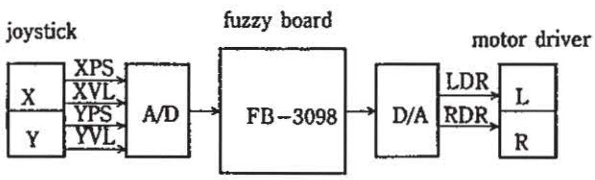

Fig. 3 Controller Blockdiagram

3. Fuzzy control

3.1 Membership functions

The pre-condition variables are the X direction component XPS, its derivative XVL, the Y direction component YPS and its derivative YVL. Fig. 4 (a) shows the membership functions. PS, NL,···etc. are fuzzy labels. In these labels, P and N are the signs. P (positive) denotes that the direction in which the pendulum is inclining is towards+X and +Y, and the velocity of the inclination is increasing. The arrangement of the fuzzy sets is concentrated at the center region so as to accomplish higher sensitivity in the neutral area of the pendulum. The conclusion variables are the left wheel drive signal (LDR) and the right wheel drive signal (RDR). Likewise, P means positive and the wheel is to be driven in the +Y directon. The membership functions of the conclusion variables, shown in Fig. 4 (b), are monotone type to ease calculation.

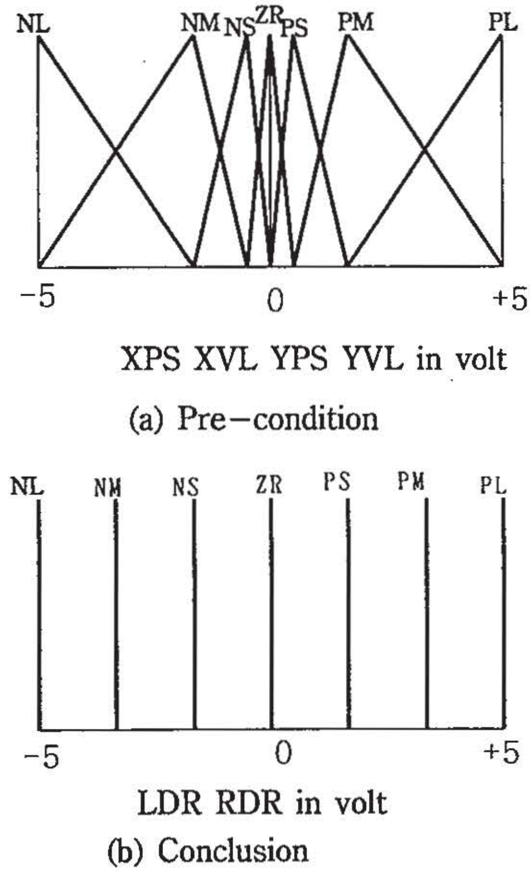

Fig. 4 Membership Function

3.2 Fuzzy rules

There are two types of fuzzy rules: for foward-backward operations and for turning operations. In actual operation, contol is done by a mixture of these rules whithin each sampling time slice.

3.2.1 Fuzzy rule table for forward-backward operatiosns

Table 2 shows the fuzzy rule table for forward-backward operations. In these cases, both wheels must turn in the same direction, therefore the sign part of the drive signals LDR and RDR are P.

3.3.2 Fuzzy rule for turn operations

The sign part of this fuzzy label is determined as follows. When the pendulum is inclining towards the +X and +Y direction, i.e. towards quadrant I, the sign part of the pre-condition variables are P for both XPS and YPS. To correct the situation, the cart must turn to match its moving direction with the inclining direction of the pendulum. To achieve this, the left wheel must be driven forward while the right wheel driven backwards, which means that the sign part is P for LDR and N for RDR. In a like manner, the sign part of LDR and RDR for the pendulum leaning towards the other quadrants are shown in Table 3. In addition to the sign, the magnitude of this fuzzy label is determined as follows. In a turn operstion, the error angle ϕ is detrmined by $\phi = \tan^{-1}\frac{XPS}{YPS}$

The variable values at the top of the triangle in each fuzzy label are used as the representative values for the XPS and YPS's membership functions. The variable values at the top of the triangle in each fuzzy label of XPS and YPS are used as the representative values for the membership functions. ϕ are calculated using the above equation. On the other hand, the labels of the conclusion membershisp function are assigned an eqally divided angle: 0°, 30°, 60° or 90°. Then the labels are determined from the above ϕ near this angle. These are summarized in Table 4.

Table 2 Fuzzy Rule Table

		YPS						
		NL	NM	NS	ZR	PS	PM	PL
	PL							
	PM							
	PS			ZR		PS		
YVL	ZR		NL		ZR		PL	
	NS			NS		ZR		
	NM							
	NL							

Table 3 Sign of Fuzzy Label

	pre−condition		conclusion	
quadrant	YPS	XPS	LDR	RDR
I	P	P	P	N
II	P	N	N	P
III	N	N	P	N
IV	N	P	N	P

Table 4 Magnitude of Fuzzy Label

				XPS			
				L	M	S	ZR
				5.0	1.6	0.5	0
			A	1	0.3	0.1	
	L	5.0	B	45	18.2	5	
			C				
			A	3.0	1	0.3	
YPS	M	1.6	B	71.5	45	71.5	
			C				
			A	10	3.3	1	0
	S	0.5	B	84.2	73.2	45	0
			C		L	S	ZR
			A				
	ZR	0	B				
			C				

A: XPS/YPS

B: $\phi = tan^{-1}\frac{XPS}{YPS}(deg)$

C: Magnitude of Label

4. Experiment results

Fig. 5 (a) shows the time evolution of XPS and YPS. From these graphs, the pendulum inclination angle θ calculated using the equation $\theta = \sqrt{XPS^2 + YPS^2}$, and error angle ϕ are plotted in Fig. 5 (b). From these graphs, it is clear that in the operation to balance the pendulum vertically, the maximum inclination angle is 5°, and the maximum error angle is 90°.

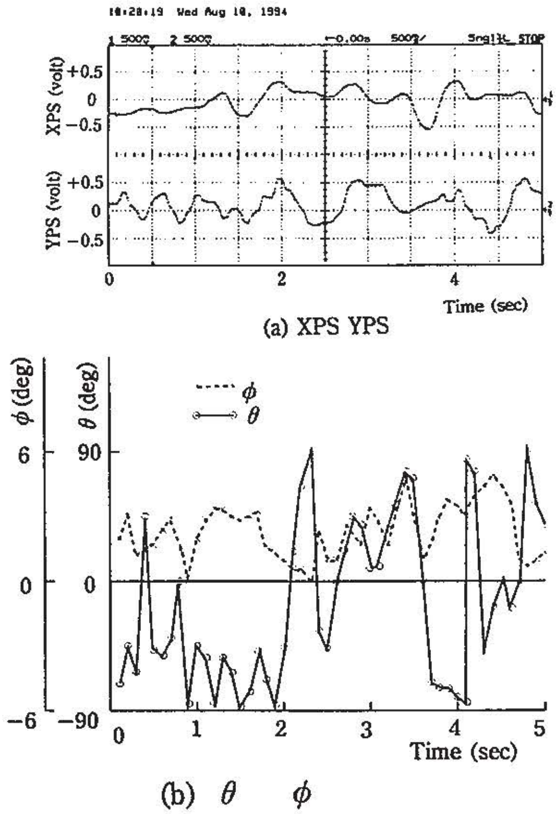

Fig. 5 Time Respnse of the System

5. Conclusion

The inverted pendulum that can incline in all directions can be made to balance on a cart that can run freely on the floor. The maximum inclination angle is 5°. In this experiment to try to balance the pendulum, the cart loses its position and is running out of the floor area because the cart does not have any position detector. A plan to incorporate a position detector or manual control is considered in future experiments.

References

1) Acheson DJ : A Pendulum Theorem, Proc R Soc Lond Ser A, Vol. 443, NO. 1917, 1993
2) T. Yamakawa : A Simple Fuzzy Computer Hardware System Employing Min & Max Operations, Preprint of 2nd ISFA Congress, Vol. 2, Japan, 1987
3) E. Ostertag, M.J. Carvalho-ostertag : Fuzzy Conrol of an Inverted Pendulum with Fuzzy Compensation of Friction Forces, INT. J. SYSTEMS SCI., Vol. 24, No. 10, 1993