

Expansion of a Wave Function in Terms of the Spherical Harmonics at a Different Site. II

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We have derived the formula for expansion of a wave function in terms of the spherical harmonics at a different site in the previous paper [1], which is referred to as I, and also have tabulated the expansions for the s- and p-wavefunctions as examples. The expansions are very useful in the calculations of the matrix elements given by the two center integral.

The results described in I are briefly reviewed here. We have treated the wave function $\psi_l(\mathbf{r})$ for the state specified by the orbital angular momentum l , and it is a linear combination of the spherical harmonics $Y_{lm}(\theta, \phi)$. Then the wave function can be written as follows,

$$\psi_l(\mathbf{r}) = Q_l(r) \sum_{\mu=-l}^l a_{l\mu} Y_{l\mu}(\theta, \phi), \quad (1)$$

where $Q_l(r)$ is the radial part and $a_{l\mu}$ is the expansion coefficient. When the wave function has the center at a position given by the vector λ , it can be denoted by $\psi_l(\mathbf{r}-\lambda)$. This $\psi_l(\mathbf{r}-\lambda)$ can be expanded around the origin using the spherical harmonics having the center there in the following form [1],

$$\psi_l(\mathbf{r}-\lambda) = \sum_{L=0}^{\infty} \sum_{M=-L}^L F_{LM}^{(l)}(\mathbf{r}, \lambda) Y_{LM}(\theta, \phi), \quad (2)$$

$$F_{LM}^{(l)}(\mathbf{r}, \lambda) = \sum_{\mu=-l}^l \sum_{\nu=-\Lambda}^{\Lambda} a_{l\mu} \exp(i(\mu-M)\alpha) r_{\mu\nu}^{(l)}(\beta) r_{M\nu}^{(L)}(\beta) f_{L\nu}^{(l)}(\mathbf{r}, \lambda), \quad (3)$$

$$f_{L\nu}^{(l)}(\mathbf{r}, \lambda) = \frac{k_{l\mu} k_{L\mu}}{\lambda r} \int_{|\lambda-r|}^{\lambda+r} dR R Q_l(R) P_l^{\mu}\left(\frac{r^2-R^2-\lambda^2}{2\lambda R}\right) P_L^{\nu}\left(\frac{r^2+\lambda^2-R^2}{2\lambda r}\right), \quad (4)$$

where λ , α and β are the polar coordinates of λ given by $\lambda = \lambda(\sin\beta\cos\alpha, \sin\beta\sin\alpha, \cos\beta)$, $r_{\mu\nu}^{(l)}(\beta) = \langle l\mu | \exp(-i\beta J_y) | l\nu \rangle$, $P_l^{\mu}(x)$ is the associated Legendre function, $k_{l\mu} = [(2l+1)/2]^{1/2} [(l-|\mu|)! / (l+|\mu|)!]^{1/2}$, $\Lambda = \min(l, L)$ and $f_{L\nu}^{(l)}(\mathbf{r}, \lambda) = f_{L|\mu|}^{(l)}(\mathbf{r}, \lambda)$. In the paper I, the coefficients of $f_{L\nu}^{(l)}(\mathbf{r}, \lambda)$ in eq. (3) are tabulated for $Y_{LM}(\theta, \phi)$ ($0 \leq L \leq 4$). In this paper, the similar tables to those for the s- and p-wavefunctions in I are made for the d- and f-wavefunctions. When we put $a_{l\mu} = \delta_{\mu m}$ in eq. (1), $\psi_l(\mathbf{r})$ becomes to the wave function $\psi_{lm}(\mathbf{r})$ with the same symmetry as $Y_{lm}(\theta, \phi)$ concerning the angular parts. Therefore, the expansion form for the $\psi_{lm}(\mathbf{r}-\lambda)$ ($m = -l, -l+1, \dots, l$) can be easily obtained by putting $a_{l\mu} = \delta_{\mu m}$ in eq. (3). The result is

$$\psi_{lm}(\mathbf{r}-\lambda) = \sum_{L=0}^{\infty} \sum_{M=-L}^L F_{LM}^{(lm)}(\alpha, \mathbf{r}, \lambda) Y_{LM}(\theta, \phi), \quad (5)$$

$$F_{LM}^{(lm)}(\alpha, \mathbf{r}, \lambda) = \exp(i(m-M)\alpha) \sum_{\nu=-\Lambda}^{\Lambda} r_{m\nu}^{(l)}(\beta) r_{M\nu}^{(L)}(\beta) f_{L\nu}^{(l)}(\mathbf{r}, \lambda). \quad (6)$$

The expansions for the d- and f-wavefunctions $\psi_{lm}(\mathbf{r}-\lambda)$ ($l=2$ for d and $l=3$ for f) are tabulated in Appendices A and B within $0 \leq L \leq 4$. The calculations in eq. (6) were performed using the tables of $r_{\mu\nu}^{(l)}(\beta)$ given in I. In the tables here listed, the expansions only for $\psi_{lm}(\mathbf{r}-\lambda)$ with $m \geq 0$ are shown since $F_{LM}^{(lm)}(\alpha, \mathbf{r}, \lambda)$ has the symmetry given by $F_{LM}^{(l,-m)}(\alpha, \mathbf{r}, \lambda) = (-1)^{m+M} F_{L,-M}^{(lm)}(-\alpha, \mathbf{r}, \lambda)$. The expansions for $\psi_{l,-m}(\mathbf{r}-\lambda)$ (m

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$\rangle 0\rangle$ can be immediately obtained from those for $\phi_{lm}(r-\lambda)$ using this symmetry. This symmetrical relation can be very easily proved using the property of the rotation matrix given by $r_{\mu-\nu}^{(l)}(\beta) = (-1)^{\mu-\nu} r_{\mu\nu}^{(l)}(\beta)$ in eq. (6)[2]. For $\phi_{l0}(r-\lambda)$, all coefficients are shown in the tables although the list for $M \geq 0$ is enough because of $F_{LM}^{(l0)}(\alpha, r, \lambda) = (-1)^M F_{L-M}^{(l0)}(-\alpha, r, \lambda)$.

Appendix A : Expansions of the d-wavefunctions $\Psi_{dm}(r-\lambda)(m=0,1,2)$

The coefficients of $f_{Lk}^{(2)}(r, \lambda)$ in $F_{LM}^{(2m)}(\alpha, r, \lambda)$ are shown for the states $Y_{LM}(\theta, \phi) (= |LM\rangle)$, in which $0 \leq k \leq L$ for $L \leq 1$ and $0 \leq k \leq 2$ for $L \geq 2$. $(l, m, n) = (\sin \beta \cos \alpha, \sin \beta \sin \alpha, \cos \beta)$: $X_{\pm} = l \pm im = \exp(\pm i\alpha) \sin \beta$: $f_{Lk} = f_{Lk}^{(2)}(r, \lambda)$

1) $\Psi_{d0}(r-\lambda)$				
<hr/>				
$L=0$	$ 0\rangle$			
f_{00}	$-(1-3n^2)/2$			
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$L=1$	$ 1\rangle$	$ 0\rangle$	$ -1\rangle$	
f_{10}	$\sqrt{2}(1-3n^2)X_-/4$	$-n(1-3n^2)/2$	$-\sqrt{2}(1-3n^2)X_+/4$	
f_{11}	$\sqrt{6}n^2X_-/2$	$\sqrt{3}n(1-n^2)$	$-\sqrt{6}n^2X_+/2$	
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$L=2$	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$	$ -1\rangle$
f_{20}	$-\sqrt{6}(1-3n^2)X_-^2/8$	$\sqrt{6}n(1-3n^2)X_-/4$	$(1-3n^2)^2/4$	$-\sqrt{6}n(1-3n^2)X_+/4$
f_{21}	$-\sqrt{6}n^2X_-^2/2$	$-\sqrt{6}n(1-2n^2)X_-/2$	$3n^2(1-n^2)$	$\sqrt{6}n(1-2n^2)X_+/2$
f_{22}	$\sqrt{6}(1+n^2)X_-^2/8$	$\sqrt{6}n(1-n^2)X_-/4$	$3(1-n^2)^2/4$	$-\sqrt{6}n(1-n^2)X_+/4$
	$ -2\rangle$			
f_{20}	$-\sqrt{6}(1-3n^2)X_+^2/8$			
f_{21}	$-\sqrt{6}n^2X_+^2/2$			
f_{22}	$\sqrt{6}(1+n^2)X_+^2/8$			
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$L=3$	$ 3\rangle$	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$
f_{30}	$\sqrt{5}(1-3n^2)X_-^3/8$	$-\sqrt{15}n(1-3n^2)X_-^2/16$	$-\sqrt{3}(1-3n^2)(1-5n^2)X_-/8$	$-n(1-3n^2)(5n^2-3)/4$
f_{31}	$3\sqrt{10}n^2X_-^3/8$	$\sqrt{15}n(1-3n^2)X_-^2/4$	$-\sqrt{6}n^2(11-15n^2)X_-/8$	$3\sqrt{2}n(1-n^2)(5n^2-1)/4$
f_{32}	$-3(1+n^2)X_-^3/8$	$-\sqrt{6}n(1-3n^2)X_-^2/8$	$-\sqrt{15}(1-n^2)(1-3n^2)X_-/8$	$3\sqrt{5}n(1-n^2)^2/4$
	$ -1\rangle$	$ -2\rangle$	$ -3\rangle$	
f_{30}	$\sqrt{3}(1-3n^2)(1-5n^2)X_+/8$	$-\sqrt{15}n(1-3n^2)X_+^2/16$	$-\sqrt{15}(1-3n^2)X_+^3/8$	
f_{31}	$\sqrt{6}n^2(11-15n^2)X_+/8$	$\sqrt{15}n(1-3n^2)X_+^2/4$	$-3\sqrt{10}n^2X_+^3/8$	
f_{32}	$\sqrt{15}(1-n^2)(1-3n^2)X_+/8$	$-\sqrt{6}n(1-3n^2)X_+^2/8$	$3(1+n^2)X_+^3/8$	
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$L=4$	$ 4\rangle$	$ 3\rangle$	$ 2\rangle$	$ 1\rangle$
f_{40}	$-\sqrt{70}(1-3n^2)X_-^4/32$	$\sqrt{35}n(1-3n^2)X_-^3/8$	$-\sqrt{10}(1-3n^2)(7n^2-1)X_-^2/16$	$\sqrt{5}n(1-3n^2)(7n^2-3)X_-/8$
f_{41}	$-\sqrt{21}n^2X_-^4/4$	$-\sqrt{42}n(1-4n^2)X_-^3/8$	$\sqrt{3}n^2(4-7n^2)X_-^2/2$	$\sqrt{6}n(3-27n^2+28n^4)X_-/8$
f_{42}	$\sqrt{42}(1+n^2)X_-^4/16$	$-\sqrt{21}n^3X_-^3/4$	$\sqrt{6}(1-6n^2+7n^4)X_-^2/8$	$-\sqrt{3}n(1-n^2)(4-7n^2)X_-/4$
	$ 0\rangle$	$ -1\rangle$	$ -2\rangle$	$ -3\rangle$
f_{40}	$-(1-3n^2)(35n^4-30n^2+3)/16$	$-\sqrt{5}n(1-3n^2)(7n^2-3)X_+/8$	$-\sqrt{10}(1-3n^2)(7n^2-1)X_+^2/16$	$-\sqrt{35}n(1-3n^2)X_+^3/8$
f_{41}	$\sqrt{30}n^2(1-n^2)(7n^2-3)/4$	$-\sqrt{6}n(3-27n^2+28n^4)X_+/8$	$\sqrt{3}n^2(4-7n^2)X_+^2/2$	$\sqrt{42}n(1-4n^2)X_+^3/8$
f_{42}	$\sqrt{15}(1-n^2)^2(7n^2-1)/8$	$\sqrt{3}n(1-n^2)(4-7n^2)X_+/4$	$\sqrt{6}(1-6n^2+7n^4)X_+^2/8$	$\sqrt{21}n^3X_+^3/4$
	$ -4\rangle$			
f_{40}	$-\sqrt{70}(1-3n^2)X_+^4/32$			
f_{41}	$-\sqrt{21}n^2X_+^4/4$			
f_{42}	$\sqrt{42}(1+n^2)X_+^4/16$			

2) $\Psi_{d1}(r-\lambda)$				
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$L=0$				
	$ 0\rangle$			
f_{00}	$-\sqrt{6}nX_+/2$			
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$L=1$				
	$ 1\rangle$	$ 0\rangle$	$ -1\rangle$	
f_{10}	$\sqrt{3}n(1-n^2)/2$	$-\sqrt{6}n^2X_+/2$	$-\sqrt{3}nX_+^2/2$	
f_{11}	n^3	$-\sqrt{2}(1-2n^2)X_+/2$	nX_+^2	
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$L=2$				
	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$	$ -1\rangle$
f_{20}	$-3n(1-n^2)X_-/4$	$3n^2(1-n^2)/2$	$\sqrt{6}n(1-3n^2)X_+/4$	$-3n^2X_+^2/2$
f_{21}	$-n^3X_-$	$(1-3n^2+4n^4)/2$	$-\sqrt{6}n(1-2n^2)X_+/2$	$-(1-4n^2)X_+^2/2$
f_{22}	$n(3+n^2)X_-/4$	$(1-n^4)/2$	$\sqrt{6}n(1-n^2)X_+/4$	$(1-n^2)X_+^2/2$
	$ -2\rangle$			
f_{20}	$-3nX_+^3/4$			
f_{21}	nX_+^3			
f_{22}	$-nX_+^3/4$			
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$L=3$				
	$ 3\rangle$	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$
f_{30}	$\sqrt{30}n(1-n^2)X_-^2/8$	$-3\sqrt{5}n^2(1-n^2)X_-/4$	$3\sqrt{2}n(1-n^2)(5n^2-1)/8$	$-\sqrt{6}n^2(5n^2-3)X_+/4$
f_{31}	$\sqrt{15}n^3X_-^2/4$	$-\sqrt{10}(1-3n^2+6n^4)X_-/8$	$n(5-16n^2+15n^4)/4$	$-\sqrt{3}(5n^2-1)(1-2n^2)X_+/4$
f_{32}	$-\sqrt{6}n(3+n^2)X_-^2/8$	$-(1-6n^2-3n^4)X_-/4$	$\sqrt{10}n(1-n^2)(3n^2+1)/8$	$\sqrt{30}n^2(1-n^2)X_+/4$
	$ -1\rangle$	$ -2\rangle$	$ -3\rangle$	
f_{30}	$-3\sqrt{2}n(5n^2-1)X_+^2/8$	$-3\sqrt{5}n^2X_+^3/4$	$-\sqrt{30}nX_+^4/8$	
f_{31}	$-3n(2-5n^2)X_+^2/4$	$-\sqrt{10}(1-6n^2)X_+^3/8$	$\sqrt{15}nX_+^4/4$	
f_{32}	$3\sqrt{10}n(1-n^2)X_+^2/8$	$(1-3n^2)X_+^3/4$	$-\sqrt{6}nX_+^4/8$	
<hr/>				
$L=4$				
	$ 4\rangle$	$ 3\rangle$	$ 2\rangle$	$ 1\rangle$
f_{40}	$-\sqrt{105}n(1-n^2)X_-^3/16$	$\sqrt{210}n^2(1-n^2)X_-^2/8$	$-\sqrt{15}n(1-n^2)(7n^2-1)X_-/8$	$\sqrt{30}n^2(1-n^2)(7n^2-3)/8$
f_{41}	$-\sqrt{14}n^3X_-^3/4$	$\sqrt{7}(1-3n^2+8n^4)X_-^2/8$	$-\sqrt{2}n(7-23n^2+28n^4)X_-/8$	$-(3-30n^2+75n^4-56n^6)/8$
f_{42}	$-\sqrt{7}n(3+n^2)X_-^3/8$	$\sqrt{14}(1-3n^2-2n^4)X_-^2/8$	$-n(4-n^2-7n^4)X_-/4$	$-\sqrt{2}(1-n^2)(1+n^2-14n^4)/8$
	$ 0\rangle$	$ -1\rangle$	$ -2\rangle$	$ -3\rangle$
f_{40}	$-\sqrt{6}n(35n^4-30n^2+3)X_+/16$	$-\sqrt{30}n^2(7n^2-3)X_+^2/8$	$-\sqrt{15}n(7n^2-1)X_+^3/8$	$-\sqrt{210}n^2X_+^4/8$
f_{41}	$-\sqrt{5}n(7n^2-3)(1-2n^2)X_+/4$	$(3-33n^2+56n^4)X_+^2/8$	$-\sqrt{2}n(9-28n^2)X_+^3/8$	$-\sqrt{7}(1-8n^2)X_+^4/8$
f_{42}	$\sqrt{10}n(7n^2-1)(1-n^2)X_+/8$	$-\sqrt{2}(1-n^2)(1-14n^2)X_+^2/8$	$n(6-7n^2)X_+^3/4$	$\sqrt{14}(1-2n^2)X_+^4/8$
	$-4\rangle$			
f_{40}	$-\sqrt{105}nX_+^5/16$			
f_{41}	$\sqrt{14}nX_+^5/4$			
f_{42}	$-\sqrt{7}nX_+^5/8$			

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3) $\Psi_{d2}(r-\lambda)$				
$L=0$	$ 0\rangle$			
f_{00}	$\sqrt{6}X_+^2/4$			
$L=1$	$ 1\rangle$	$ 0\rangle$	$ -1\rangle$	
f_{10}	$\sqrt{3}(1-n^2)X_+/4$	$\sqrt{6}nX_+^2/4$	$\sqrt{3}X_+^3/4$	
f_{11}	$-(1+n^2)X_+/2$	$-\sqrt{2}nX_+^2/2$	$-X_+^3/2$	
$L=2$	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$	$ -1\rangle$
f_{20}	$3(1-n^2)^2/8$	$-3n(1-n^2)X_+/4$	$-\sqrt{6}(1-3n^2)X_+^2/8$	$3nX_+^3/4$
f_{21}	$(1-n^2)(1+n^2)/2$	$-n^3X_+$	$-\sqrt{6}n^2X_+^2/2$	$-nX_+^3$
f_{22}	$(1+6n^2+n^4)/8$	$n(3+n^2)X_+/4$	$\sqrt{6}(1+n^2)X_+^2/8$	$nX_+^3/4$
	$ -2\rangle$			
f_{20}	$3X_+^4/8$			
f_{21}	$-X_+^4/2$			
f_{22}	$X_+^4/8$			
$L=3$	$ 3\rangle$	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$
f_{30}	$-\sqrt{30}(1-n^2)^2X_-/16$	$3\sqrt{5}n(1-n^2)^2/8$	$-3\sqrt{2}(1-n^2)(5n^2-1)X_+/16$	$\sqrt{6}n(5n^2-3)X_+^2/8$
f_{31}	$-\sqrt{15}(1-n^2)(1+n^2)X_-/8$	$\sqrt{10}n(1-n^2)(1+3n^2)/8$	$(1+6n^2-15n^4)X_+/8$	$-\sqrt{3}n(5n^2-1)X_+^2/4$
f_{32}	$-\sqrt{6}(1+6n^2+n^4)X_-/16$	$-n(5-10n^2-3n^4)/8$	$-\sqrt{10}(1-6n^2-3n^4)X_+/16$	$\sqrt{30}n(1+n^2)X_+^2/8$
	$ -1\rangle$	$ -2\rangle$	$ -3\rangle$	
f_{30}	$3\sqrt{2}(5n^2-1)X_+^3/16$	$3\sqrt{5}nX_+^4/4$	$\sqrt{30}X_+^5/16$	
f_{31}	$(1-15n^2)X_+^3/8$	$-3\sqrt{10}nX_+^4/8$	$\sqrt{15}X_+^5/8$	
f_{32}	$\sqrt{10}(1+3n^2)X_+^3/16$	$3nX_+^4/8$	$\sqrt{6}X_+^5/16$	
$L=4$	$ 4\rangle$	$ 3\rangle$	$ 2\rangle$	$ 1\rangle$
f_{40}	$\sqrt{105}(1-n^2)^2X_-^2/32$	$-\sqrt{210}n(1-n^2)^2X_-/16$	$\sqrt{15}(1-n^2)^2(7n^2-1)/16$	$-\sqrt{30}n(1-n^2)(7n^2-3)X_+/16$
f_{41}	$\sqrt{14}(1-n^2)(1+n^2)X_-^2/8$	$-\sqrt{7}n(1-n^2)(1+2n^2)X_-/4$	$-\sqrt{2}(1-n^2)(1+n^2-14n^4)/8$	$n^3(5-7n^2)X_+/2$
f_{42}	$\sqrt{7}(1+6n^2+n^4)X_-^2/16$	$\sqrt{14}n(1-4n^2-n^4)X_-/8$	$(1-15n^2+15n^4+7n^6)/8$	$-\sqrt{2}n(5-10n^2-7n^4)X_+/8$
	$ 0\rangle$	$ -1\rangle$	$ -2\rangle$	$ -3\rangle$
f_{40}	$\sqrt{6}(35n^4-30n^2+3)X_+^2/32$	$\sqrt{30}n(7n^2-3)X_+^3/16$	$\sqrt{15}(7n^2-1)X_+^4/16$	$\sqrt{210}nX_+^5/16$
f_{41}	$-\sqrt{5}n^2(7n^2-3)X_+^2/4$	$n(3-14n^2)X_+^3/4$	$\sqrt{2}(1-14n^2)X_+^4/8$	$-\sqrt{7}nX_+^5/2$
f_{42}	$\sqrt{10}(1+n^2)(7n^2-1)X_+^2/16$	$\sqrt{2}n(3+7n^2)X_+^3/8$	$(1+7n^2)X_+^4/8$	$\sqrt{14}nX_+^5/8$
	$ -4\rangle$			
f_{40}	$\sqrt{105}X_+^6/32$			
f_{41}	$-\sqrt{14}X_+^6/8$			
f_{42}	$\sqrt{7}X_+^6/16$			

Appendix B : Expansions of the f-wavefunctions $\Psi_{fm}(r-\lambda)(m=0,1,2,3)$

The coefficients of $f_{Lk}^{(3)}(r, \lambda)$ in $F_{LM}^{(3m)}(\alpha, r, \lambda)$ are shown for the states $Y_{LM}(\theta, \phi) (= |LM\rangle)$, in which $0 \leq k \leq L$ for $L \leq 2$ and $0 \leq k \leq 3$ for $L \geq 3$, $(l, m, n) = (\sin \beta \cos \alpha, \sin \beta \sin \alpha, \cos \beta)$: $X_{\pm} = l \pm im = \exp(\pm i\alpha) \sin \beta$: $f_{Lk} = f_{Lk}^{(3)}(r, \lambda)$

1) $\Psi_{f0}(r-\lambda)$				
$L=0$	$ 0\rangle$			
f_{00}	$n(5n^2-3)/2$			
$L=1$	$ 1\rangle$	$ 0\rangle$	$ -1\rangle$	
f_{10}	$-\sqrt{2}n(5n^2-3)X_-/4$	$n^2(5n^2-3)/2$	$\sqrt{2}n(5n^2-3)X_+/4$	
f_{11}	$\sqrt{3}n(5n^2-1)X_-/4$	$\sqrt{6}(1-n^2)(5n^2-1)/4$	$-\sqrt{3}n(5n^2-1)X_+/4$	
$L=2$	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$	$ -1\rangle$
f_{20}	$\sqrt{6}n(5n^2-3)X_-^2/8$	$-\sqrt{6}n^2(5n^2-3)X_-/4$	$n(5n^2-3)(3n^2-1)/4$	$\sqrt{6}n^2(5n^2-3)X_+/4$
f_{21}	$-\sqrt{3}n(5n^2-1)X_-^2/4$	$\sqrt{3}(5n^2-1)(2n^2-1)X_-/4$	$3\sqrt{2}n(1-n^2)(5n^2-1)/4$	$-\sqrt{3}(5n^2-1)(2n^2-1)X_+/4$
f_{22}	$\sqrt{30}n(n^2+1)X_-^2/8$	$\sqrt{30}n^2(1-n^2)X_-/4$	$3\sqrt{5}n(1-n^2)^2/4$	$-\sqrt{30}n^2(1-n^2)X_+/4$
	$ -2\rangle$			
f_{20}	$\sqrt{6}n(5n^2-3)X_+^2/8$			
f_{21}	$-\sqrt{3}n(5n^2-1)X_+^2/4$			
f_{22}	$\sqrt{30}n(n^2+1)X_+^2/8$			
$L=3$	$ 3\rangle$	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$
f_{30}	$-\sqrt{5}n(5n^2-3)X_-^3/8$	$\sqrt{30}n^2(5n^2-3)X_-^2/8$	$-\sqrt{3}n(5n^2-1)(5n^2-3)X_-/8$	$n^2(5n^2-3)^2/4$
f_{31}	$3\sqrt{5}n(5n^2-1)X_-^3/16$	$\sqrt{30}(5n^2-1)(1-3n^2)X_-^2/16$	$\sqrt{3}n(5n^2-1)(15n^2-11)X_-/16$	$\sqrt{3}(1-n^2)(5n^2-1)^2/8$
f_{32}	$-3\sqrt{5}n(1+n^2)X_-^3/8$	$\sqrt{30}n^2(3n^2-1)X_-^2/8$	$5\sqrt{3}n(1-n^2)(3n^2-1)X_-/8$	$15n^2(1-n^2)^2/8$
f_{33}	$\sqrt{5}n(3+n^2)X_-^3/16$	$\sqrt{30}(1-n^4)X_-^2/16$	$5\sqrt{3}n(1-n^2)^2X_-/16$	$5(1-n^2)^3/8$
	$ -1\rangle$	$ -2\rangle$	$ -3\rangle$	
f_{30}	$\sqrt{3}n(5n^2-1)(5n^2-3)X_+/8$	$\sqrt{30}n^2(5n^2-3)X_+^2/8$	$\sqrt{5}n(5n^2-3)X_+^3/8$	
f_{31}	$-\sqrt{3}n(5n^2-1)(15n^2-11)X_+/16$	$\sqrt{30}(5n^2-1)(1-3n^2)X_+^2/16$	$-3\sqrt{5}n(5n^2-1)X_+^3/16$	
f_{32}	$-5\sqrt{3}n(1-n^2)(3n^2-1)X_+/8$	$\sqrt{30}n^2(3n^2-1)X_+^2/8$	$3\sqrt{5}n(1+n^2)X_+^3/8$	
f_{33}	$-5\sqrt{3}n(1-n^2)^2X_+/16$	$\sqrt{30}(1-n^4)X_+^2/16$	$-\sqrt{5}n(3+n^2)X_+^3/16$	
$L=4$	$ 4\rangle$	$ 3\rangle$	$ 2\rangle$	$ 1\rangle$
f_{40}	$\sqrt{70}n(5n^2-3)X_-^4/32$	$-\sqrt{35}n^2(5n^2-3)X_-^3/8$	$\sqrt{10}n(5n^2-3)(7n^2-1)X_-^2/16$	$-\sqrt{5}n^2(5n^2-3)(7n^2-3)X_-/8$
f_{41}	$-\sqrt{42}n(5n^2-1)X_-^4/16$	$\sqrt{21}(5n^2-1)(4n^2-1)X_-^3/16$	$\sqrt{6}n(5n^2-1)(4-7n^2)X_-^2/8$	$\sqrt{3}(5n^2-1)(3-27n^2+28n^4)X_-/16$
f_{42}	$\sqrt{210}n(1+n^2)X_-^4/16$	$-\sqrt{105}n^4X_-^3/4$	$\sqrt{30}n(1-6n^2+7n^4)X_-^2/8$	$\sqrt{15}n^2(1-n^2)(7n^2-4)X_-/4$
f_{43}	$-\sqrt{10}n(3+n^2)X_-^4/16$	$\sqrt{5}(4n^4+3n^2-3)X_-^3/16$	$\sqrt{70}(1-n^2)n^3X_-^2/8$	$\sqrt{35}(1-n^2)^2(4n^2-1)X_-/16$
	$ 0\rangle$	$ -1\rangle$	$ -2\rangle$	$ -3\rangle$
f_{40}	$n(5n^2-3)(35n^4-30n^2+3)/16$	$\sqrt{5}n^2(5n^2-3)(7n^2-3)X_+/8$	$\sqrt{10}n(5n^2-3)(7n^2-1)X_+^2/16$	$\sqrt{35}n^2(5n^2-3)X_+^3/8$
f_{41}	$\sqrt{15}n(1-n^2)(5n^2-1)(7n^2-3)/8$	$-\sqrt{3}(5n^2-1)(3-27n^2+28n^4)X_+/16$	$\sqrt{6}n(5n^2-1)(4-7n^2)X_+^2/8$	$-\sqrt{21}(5n^2-1)(4n^2-1)X_+^3/16$
f_{42}	$5\sqrt{3}n(1-n^2)^2(7n^2-1)/8$	$-\sqrt{15}n^2(1-n^2)(7n^2-4)X_+/4$	$\sqrt{30}n(1-6n^2+7n^4)X_+^2/8$	$\sqrt{105}n^4X_+^3/4$
f_{43}	$5\sqrt{7}n(1-n^2)^3/8$	$-\sqrt{35}(1-n^2)^2(4n^2-1)X_+/16$	$\sqrt{70}(1-n^2)n^3X_+^2/8$	$-\sqrt{5}(4n^4+3n^2-3)X_+^3/16$
	$ -4\rangle$			
f_{40}	$\sqrt{70}n(5n^2-3)X_+^4/32$			
f_{41}	$-\sqrt{42}n(5n^2-1)X_+^4/16$			
f_{42}	$\sqrt{210}n(1+n^2)X_+^4/16$			
f_{43}	$-\sqrt{10}n(3+n^2)X_+^4/16$			

Expansion of a Wave Function in Terms of the Spherical Harmonics

2) $\Psi_{f1}(r-\lambda)$				
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$L=0$		$ 0\rangle$		
	f_{00}	$-\sqrt{3}(5n^2-1)X_+/4$		
$L=1$		$ 1\rangle$	$ 0\rangle$	$ -1\rangle$
	f_{10}	$\sqrt{6}(1-n^2)(5n^2-1)/8$	$-\sqrt{3}n(5n^2-1)X_+/4$	$-\sqrt{6}(5n^2-1)X_+^2/8$
	f_{11}	$(15n^4-6n^2-1)/8$	$\sqrt{2}n(15n^2-11)X_+/8$	$(15n^2-1)X_+^2/8$
$L=2$		$ 2\rangle$	$ 1\rangle$	$ 0\rangle$
	f_{20}	$-3\sqrt{2}(1-n^2)(5n^2-1)X_-/16$	$3\sqrt{2}n(1-n^2)(5n^2-1)/8$	$\sqrt{3}(5n^2-1)(1-3n^2)X_+/8$
	f_{21}	$(1+6n^2-15n^4)X_-/8$	$n(15n^4-16n^2+5)/4$	$\sqrt{6}n^2(15n^2-11)X_+/8$
	f_{22}	$\sqrt{10}(3n^4+6n^2-1)X_-/16$	$\sqrt{10}n(1-n^2)(3n^2+1)/8$	$\sqrt{15}(1-n^2)(3n^2-1)X_+/8$
		$ -2\rangle$		$ -1\rangle$
	f_{20}	$-3\sqrt{2}(5n^2-1)X_+^3/16$		$-3\sqrt{2}n(5n^2-1)X_+^2/8$
	f_{21}	$(15n^2-1)X_+^3/8$		$3n(5n^2-2)X_+^2/4$
	f_{22}	$-\sqrt{10}(3n^2+1)X_+^3/16$		$-3\sqrt{10}n(1-n^2)X_+^2/8$
		$ 3\rangle$	$ 2\rangle$	$ 1\rangle$
	f_{30}	$\sqrt{15}(1-n^2)(5n^2-1)X_-^2/16$	$-3\sqrt{10}n(1-n^2)(5n^2-1)X_-/16$	$3(1-n^2)(5n^2-1)^2/16$
$L=3$	f_{31}	$-\sqrt{15}(1+6n^2-15n^4)X_-^2/32$	$-\sqrt{10}n(9-38n^2+45n^4)X_-/32$	$(1+11n^2-305n^4+225n^6)/32$
	f_{32}	$\sqrt{15}(1-6n^2-3n^4)X_-^2/16$	$\sqrt{10}n(9n^4+2n^2-3)X_-/16$	$5(1-n^2)(1-2n^2+9n^4)/16$
	f_{33}	$\sqrt{15}(1+6n^2+n^4)X_-^2/32$	$3\sqrt{10}n(1-n^2)(n^2+3)X_-/32$	$15(1-n^2)(1-n^4)/32$
		$ -1\rangle$	$ -2\rangle$	$ -3\rangle$
	f_{30}	$-3\sqrt{10}(5n^2-1)^2X_+^2/16$	$-3\sqrt{10}n(5n^2-1)X_+^3/16$	$-\sqrt{15}(5n^2-1)X_+^4/16$
	f_{31}	$(1-130n^2+225n^4)X_+^2/32$	$\sqrt{10}n(45n^2-13)X_+^3/32$	$\sqrt{15}(15n^2-1)X_+^4/32$
	f_{32}	$5(1-n^2)(9n^2-1)X_+^2/16$	$\sqrt{10}n(5-9n^2)X_+^3/16$	$-\sqrt{15}(3n^2+1)X_+^4/16$
	f_{33}	$15(1-n^2)^2X_+^2/32$	$-3\sqrt{10}n(1-n^2)X_+^3/32$	$\sqrt{15}(1+n^2)X_+^4/32$
		$ 4\rangle$	$ 3\rangle$	$ 2\rangle$
	f_{40}	$-\sqrt{210}(1-n^2)(5n^2-1)X_-^3/64$	$\sqrt{105}n(1-n^2)(5n^2-1)X_-^2/16$	$-\sqrt{30}(1-n^2)(5n^2-1)(7n^2-1)X_-/32$
$L=4$	f_{41}	$\sqrt{14}(1+6n^2-15n^4)X_-^3/32$	$\sqrt{7}n(2-11n^2+15n^4)X_-^2/8$	$-\sqrt{2}(1+76n^2-239n^4+210n^6)X_-/32$
	f_{42}	$\sqrt{70}(3n^4+6n^2-1)X_-^3/32$	$\sqrt{35}n(1-2n^2-3n^4)X_-^2/8$	$\sqrt{10}(21n^6-11n^4-n^2-1)X_-/16$
	f_{43}	$-\sqrt{30}(1+6n^2+n^4)X_-^3/32$	$\sqrt{15}n(n^4+3n^2-2)X_-^2/8$	$\sqrt{210}(1-n^2)(2n^4+3n^2-1)X_-/32$
		$ 0\rangle$	$ -1\rangle$	$ -2\rangle$
	f_{40}	$-\sqrt{3}(5n^2-1)(35n^4-30n^2+3)X_+/32$	$-\sqrt{15}n(5n^2-1)(7n^2-3)X_+^2/16$	$-\sqrt{30}(5n^2-1)(7n^2-1)X_+^3/32$
	f_{41}	$\sqrt{5}n^2(7n^2-3)(15n^2-11)X_+/16$	$n(9-82n^2+105n^4)X_+^2/8$	$\sqrt{2}(1-99n^2+210n^4)X_+^3/32$
	f_{42}	$5(1-n^2)(7n^2-1)(3n^2-1)X_+/16$	$\sqrt{5}n(1-n^2)(21n^2-5)X_+^2/8$	$-\sqrt{10}(1-14n^2+21n^4)X_+^3/16$
	f_{43}	$5\sqrt{21}n^2(1-n^2)^2X_+/16$	$\sqrt{105}n(1-n^2)^2X_+^2/8$	$\sqrt{210}(1-n^2)(1-2n^2)X_+^3/32$
		$ -4\rangle$		$ -3\rangle$
	f_{40}	$-\sqrt{210}(5n^2-1)X_+^5/64$		$-\sqrt{105}n(5n^2-1)X_+^4/16$
$L=4$	f_{41}	$\sqrt{14}(15n^2-1)X_+^5/32$		$\sqrt{7}n(30n^2-7)X_+^4/16$
	f_{42}	$-\sqrt{70}(3n^2+1)X_+^5/32$		$\sqrt{35}n(1-3n^2)X_+^4/8$
	f_{43}	$\sqrt{30}(1+n^2)X_+^5/32$		$\sqrt{15}n(2n^2-1)X_+^4/16$

3) $\Psi_{f2}(r-\lambda)$				
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$L=0$	$ 0\rangle$			
f_{00}	$\sqrt{30n}X_+^2/4$			
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$L=1$	$ 1\rangle$	$ 0\rangle$	$ -1\rangle$	
f_{10}	$-\sqrt{15n}(1-n^2)X_+/4$	$\sqrt{30n^2}X_+^2/4$	$\sqrt{15n}X_+^3/4$	
f_{11}	$-\sqrt{10n}(3n^2+1)X_+/8$	$\sqrt{5}(1-3n^2)X_+^2/4$	$-3\sqrt{10n}X_+^3/8$	
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$L=2$	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$	$ -1\rangle$
f_{20}	$3\sqrt{5n}(1-n^2)^2/8$	$-3\sqrt{5n^2}(1-n^2)X_+/4$	$\sqrt{30n}(3n^2-1)X_+^2/8$	$3\sqrt{5n^2}X_+^3/4$
f_{21}	$\sqrt{10n}(1-n^2)(3n^2+1)/8$	$-\sqrt{10}(1-3n^2+6n^4)X_+/8$	$\sqrt{15n}(1-3n^2)X_+^2/4$	$\sqrt{10}(1-6n^2)X_+^3/8$
f_{22}	$n(3n^4+10n^2-5)/8$	$(3n^4+3n^2-2)X_+/4$	$\sqrt{6n}(3n^2-1)X_+^2/8$	$(3n^2-2)X_+^3/4$
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	$ -2\rangle$			
f_{20}	$\sqrt{30n}X_+^4/16$			
f_{21}	$-3\sqrt{10n}X_+^4/8$			
f_{22}	$3nX_+^4/8$			
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$L=3$	$ 3\rangle$	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$
f_{30}	$-5\sqrt{6n}(1-n^2)^2X_+/16$	$15n^2(1-n^2)^2/8$	$3\sqrt{10n}(1-n^2)(1-5n^2)X_+/16$	$\sqrt{30n^2}(5n^2-3)X_+^2/8$
f_{31}	$-5\sqrt{6n}(1-n^2)(3n^2+1)X_+/32$	$5(1-n^2)(1-2n^2+9n^4)/16$	$-\sqrt{10n}(9-38n^2+45n^4)X_+/32$	$\sqrt{30}(5n^2-1)(1-3n^2)X_+^2/16$
f_{32}	$\sqrt{6n}(5-10n^2-3n^4)X_+/16$	$(4-15n^2+10n^4+9n^6)/8$	$\sqrt{10n}(9n^4+2n^2-3)X_+/16$	$\sqrt{30n^2}(3n^2-1)X_+^2/8$
f_{33}	$\sqrt{6n}(5+10n^2+n^4)X_+/32$	$3(1-n^2)(n^4+6n^2+1)/16$	$3\sqrt{10n}(1-n^2)(n^2+3)X_+/32$	$\sqrt{30}(1-n^4)X_+^2/16$
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	$ -1\rangle$	$ -2\rangle$	$ -3\rangle$	
f_{30}	$3\sqrt{10n}(5n^2-1)X_+^3/16$	$15n^2X_+^4/8$	$5\sqrt{6n}X_+^5/16$	
f_{31}	$\sqrt{10n}(13-45n^2)X_+^3/32$	$5(1-9n^2)X_+^4/16$	$-15\sqrt{6n}X_+^5/32$	
f_{32}	$\sqrt{10n}(9n^2-5)X_+^3/16$	$(9n^2-4)X_+^4/8$	$3\sqrt{6n}X_+^5/16$	
f_{33}	$3\sqrt{10n}(1-n^2)X_+^3/32$	$3(1-n^2)X_+^4/16$	$-\sqrt{6n}X_+^5/32$	
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$L=4$	$ 4\rangle$	$ 3\rangle$	$ 2\rangle$	$ 1\rangle$
f_{40}	$5\sqrt{21n}(1-n^2)^2X_+^2/32$	$-5\sqrt{42n^2}(1-n^2)^2X_+/16$	$5\sqrt{3n}(1-n^2)^2(7n^2-1)/16$	$-5\sqrt{6n^2}(1-n^2)(7n^2-3)X_+/16$
f_{41}	$\sqrt{35n}(1-n^2)(3n^2+1)X_+^2/16$	$-\sqrt{70}(1-n^2)(1-n^2+12n^4)X_+/32$	$3\sqrt{5n}(1-n^2)(1-4n^2+7n^4)/8$	$\sqrt{10}(3-30n^2+95n^4-84n^6)X_+/32$
f_{42}	$\sqrt{7n}(3n^4-26n^2-5)X_+^2/16$	$-\sqrt{14}(1-5n^2+5n^4+3n^6)X_+/8$	$n(9-25n^2+3n^4+21n^6)/8$	$\sqrt{2}(1-10n^2-10n^4+42n^6)X_+/16$
f_{43}	$-\sqrt{3n}(n^4+10n^2+5)X_+^2/16$	$-\sqrt{6}(3+10n^2-25n^4-4n^6)X_+/32$	$\sqrt{21n}(1-n^2)(n^4+4n^2-1)/8$	$-\sqrt{42}(1-n^2)(1-9n^2+4n^4)X_+/32$
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	$ 0\rangle$	$ -1\rangle$	$ 2\rangle$	$ -3\rangle$
f_{40}	$\sqrt{30n}(35n^4-30n^2+3)X_+^2/32$	$5\sqrt{6n^2}(7n^2-3)X_+^3/16$	$5\sqrt{3n}(7n^2-1)X_+^4/16$	$5\sqrt{42n^2}X_+^5/16$
f_{41}	$5\sqrt{2n}(7n^2-3)(1-3n^2)X_+^2/16$	$-3\sqrt{10}(1-13n^2+28n^4)X_+^3/32$	$\sqrt{5n}(5-21n^2)X_+^4/8$	$\sqrt{70}(1-12n^2)X_+^5/32$
f_{42}	$\sqrt{10n}(7n^2-1)(3n^2-1)X_+^2/16$	$\sqrt{2}(1-12n^2+21n^4)X_+^3/8$	$n(21n^2-11)X_+^4/8$	$\sqrt{14}(3n^2-1)X_+^5/8$
f_{43}	$\sqrt{210n}(1-n^4)X_+^2/16$	$\sqrt{42}(1-n^2)(4n^2+1)X_+^3/32$	$\sqrt{21n}(1-n^2)X_+^4/8$	$\sqrt{6}(3-4n^2)X_+^5/32$
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	$ -4\rangle$			
f_{40}	$5\sqrt{21n}X_+^6/32$			
f_{41}	$-3\sqrt{35n}X_+^6/16$			
f_{42}	$3\sqrt{7n}X_+^6/16$			
f_{43}	$-\sqrt{3n}X_+^6/16$			

Expansion of a Wave Function in Terms of the Spherical Harmonics

3) $\Psi_{f3}(r-\lambda)$				
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$L=0$	$ 0\rangle$			
f_{00}	$-\sqrt{5}X_+^3/4$			
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$L=1$	$ 1\rangle$	$ 0\rangle$	$ -1\rangle$	
f_{10}	$\sqrt{10}(1-n^2)X_+^2/8$	$-\sqrt{5}nX_+^3/4$	$-\sqrt{10}X_+^4/8$	
f_{11}	$\sqrt{15}(1+n^2)X_+^2/8$	$\sqrt{30}nX_+^3/8$	$\sqrt{15}X_+^4/8$	
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$L=2$	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$	$ -1\rangle$
f_{20}	$-\sqrt{30}(1-n^2)^2X_+/16$	$\sqrt{30}n(1-n^2)X_+^2/8$	$\sqrt{5}(1-3n^2)X_+^3/8$	$-\sqrt{30}nX_+^4/8$
f_{21}	$-\sqrt{15}(1-n^4)X_+/8$	$\sqrt{15}n(1+2n^2)X_+^2/8$	$3\sqrt{10}n^2X_+^3/8$	$\sqrt{15}nX_+^4/4$
f_{22}	$-\sqrt{6}(1+6n^2+n^4)X_+/16$	$\sqrt{6}n(3+n^2)X_+^2/8$	$-3(1+n^2)X_+^3/8$	$-\sqrt{6}nX_+^4/8$
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	$ -2\rangle$			
f_{20}	$-\sqrt{30}X_+^5/16$			
f_{21}	$\sqrt{15}X_+^5/8$			
f_{22}	$-\sqrt{6}X_+^5/16$			
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$L=3$	$ 3\rangle$	$ 2\rangle$	$ 1\rangle$	$ 0\rangle$
f_{30}	$5(1-n^2)^3/16$	$-5\sqrt{6}n(1-n^2)^2X_+/16$	$\sqrt{15}(1-n^2)(5n^2-1)X_+^2/16$	$-\sqrt{5}n(5n^2-3)X_+^3/8$
f_{31}	$15(1-n^2)(1-n^4)/32$	$-5\sqrt{6}n(1-n^2)(1+3n^2)X_+/32$	$-\sqrt{15}(1+6n^2-15n^4)X_+^2/32$	$3\sqrt{5}n(5n^2-1)X_+^3/16$
f_{32}	$3(1-n^2)(1+6n^2+n^4)/16$	$\sqrt{6}n(5-10n^2-3n^4)X_+/16$	$\sqrt{15}(1-6n^2-3n^4)X_+^2/16$	$-3\sqrt{5}n(1+n^2)X_+^3/8$
f_{33}	$(1+15n^2+15n^4+n^6)/32$	$\sqrt{6}n(5+10n^2+n^4)X_+/32$	$\sqrt{15}(1+6n^2+n^4)X_+^2/32$	$\sqrt{5}n(n^2+3)X_+^3/16$
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	$ -1\rangle$	$ -2\rangle$	$ -3\rangle$	
f_{30}	$-\sqrt{15}(5n^2-1)X_+^4/16$	$-5\sqrt{6}nX_+^5/16$	$-5X_+^6/16$	
f_{31}	$\sqrt{15}(15n^2-1)X_+^4/32$	$15\sqrt{6}nX_+^5/32$	$15X_+^6/32$	
f_{32}	$-\sqrt{15}(3n^2+1)X_+^4/16$	$-3\sqrt{6}nX_+^5/16$	$-3X_+^6/16$	
f_{33}	$\sqrt{15}(1+n^2)X_+^4/32$	$\sqrt{6}nX_+^5/32$	$X_+^6/32$	
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$L=4$	$ 4\rangle$	$ 3\rangle$	$ 2\rangle$	$ 1\rangle$
f_{40}	$-5\sqrt{14}(1-n^2)^3X_+/64$	$5\sqrt{7}n(1-n^2)^3/16$	$-5\sqrt{2}(1-n^2)^2(7n^2-1)X_+/32$	$5n(1-n^2)(7n^2-3)X_+^2/16$
f_{41}	$-\sqrt{210}(1-n^2)(1-n^4)X_+/32$	$\sqrt{105}n(2n^2+1)(1-n^2)^2/16$	$\sqrt{30}(1-n^2)(1+n^2-14n^4)X_+/32$	$\sqrt{15}n^3(7n^2-5)X_+^2/8$
f_{42}	$-\sqrt{42}(1-n^2)(1+6n^2+n^4)X_+/32$	$\sqrt{21}n(1-n^2)(n^4+4n^2-1)/8$	$-\sqrt{6}(1-15n^2+15n^4+7n^6)X_+/16$	$\sqrt{3}n(5+4n^2-7n^4)X_+^2/8$
f_{43}	$-\sqrt{2}(1+15n^2+15n^4+n^6)X_+/32$	$n(2n^6+21n^4-6)/16$	$\sqrt{14}(2n^6+15n^4-1)X_+/32$	$\sqrt{7}n^3(5+n^2)X_+^2/8$
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	$ 0\rangle$	$ -1\rangle$	$ -2\rangle$	$ -3\rangle$
f_{40}	$-\sqrt{5}(35n^4-30n^2+3)X_+^3/32$	$-5n(7n^2-3)X_+^4/16$	$-5\sqrt{2}(7n^2-1)X_+^5/32$	$-5\sqrt{7}nX_+^6/16$
f_{41}	$5\sqrt{3}n^2(7n^2-3)X_+^3/16$	$\sqrt{15}n(14n^2-3)X_+^4/16$	$\sqrt{30}(14n^2-1)X_+^5/32$	$\sqrt{105}nX_+^6/8$
f_{42}	$-\sqrt{15}(7n^2-1)(n^2+1)X_+^3/16$	$-\sqrt{3}n(7n^2+3)X_+^4/8$	$-\sqrt{6}(7n^2+1)X_+^5/16$	$-\sqrt{21}nX_+^6/8$
f_{43}	$\sqrt{35}n^2(n^2+3)X_+^3/16$	$\sqrt{7}n(2n^2+3)X_+^4/16$	$\sqrt{14}(2n^2+1)X_+^5/32$	$nX_+^6/8$
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	$ -4\rangle$			
f_{40}	$-5\sqrt{14}X_+^7/64$			
f_{41}	$\sqrt{210}X_+^7/32$			
f_{42}	$-\sqrt{42}X_+^7/32$			
f_{43}	$\sqrt{2}X_+^7/32$			

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