# Experiments and Analysis of Negative Resistance

Takeji Sato

(Received on 31 October, 1979)

A negative resistance can be thought of as an energy source that puts electric energy back into the system. In general a two-terminal active network as a switching circuit exhibits a negative resistance over a portion of the range which is across the energy-storage element terminal. But a volt-ampere characteristic over the input terminals of switching circuit also arises a negative-resistance.

In this paper, the above driving-point impedance exhibiting negative resistance characteristics is analyzed logically making use of the equivalent circuits and checked by the experiments.

#### 1. Introduction

There are basically two types of negative resistance characteristics a device, oscillator, and multivibrator might have. These are either short-circuit stable (N-type) or open circuit stable (S-type).

The physical meaning of a negative resistance is that in effect the above resistance cancels out the resistance losses of the circuit. Oscillator and multivibrator have the negative resistance which cannot be physically separated, although the tunnel diode and unijunction transistor, etc. are negative resistance devices. The object here is to describe the circuit analysis and experimental results of a bistable multivibrator as a switching circuit.

# 2. Analysis of Switching Circuit

In the cases of the saturation and cut off states, almost equivalent circuit using switches are introduced in the analysis of the transistor circuit, and we use the equivalent circuit which is denoted by r parameters in the active state. As the transistor Tohshiba 2SB-56 is selected, whose input impedance is  $h_{ie} = 4KQ$ , and current gain is  $h_{fe} = 80$  in our measurements.

We see that this circuit shown in Fig. 1 is two stages direct coupled amplifier of which the part

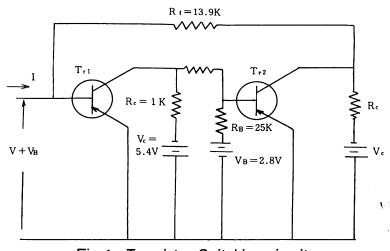


Fig. 1 Transistor Switching circuit

of the output applied to the input terminals through the feedback resistance R<sub>f</sub>. Viewed in certain aspect, the circuit composes a bistable multivibrator.

As the first step, we will divide the permissible circuit states into five regions, and the possible regions are :

Region 1 Tr<sub>1</sub> saturated and Tr<sub>2</sub> off

Region 2 Tr<sub>1</sub> active and Tr<sub>2</sub> off

Region 3 Both transtor in their active regions

Region 4 Tr<sub>1</sub> active and Tr<sub>2</sub> saturated

Region 5 Tr<sub>1</sub> off and Tr<sub>2</sub> saturated

(1) Region 1 ··· Although here equivalent circuit of Fig. 2 logically becomes the short circuit and hence the current goes through infinity, as the transistor has really an input resistance, the equation of I, then, is given by

$$I = \frac{V + V_B}{R} \tag{1-1}$$

From Eq. (1-1) we find that in this zone the input resistance of the active circuit has some positive value.

(2) Region 2 ..... From the circuit's nodal equations, referring to Fig.3, we have the next Eqs.

$$\frac{V_{c1} + V_{c}}{R_{c}} = \beta I_{B1} + \frac{V_{B} - v_{c1}}{R_{B} + R_{f}} \quad (2-1)$$

$$\frac{v_{B1} + V_c}{R_f + R_c} = I_B + I \tag{2-2}$$

$$(\beta + 1)I_{B1}r_c + I_{B1}r_b = -v_{B1}$$
 (2-3)

$$v_{b1} = V_B + V$$
 (2-4)

The base current is then found to be

$$I_B = -\frac{1}{r}(V + V_B)$$

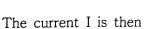
where

$$r = (\beta + 1) r_e + r_b$$

and

$$\frac{V_B + V + V_c}{R_f + R_c} = -\frac{1}{r}(V_B + V) + 1$$

(2-5)



$$I = \left(\frac{1}{r} + \frac{1}{R_f + R_c}\right) V + \left(\frac{1}{r} + \frac{1}{R_f + R_c}\right) V_B + \frac{V_c}{R_f + R_c}$$

Making the actual values substitutions in the above Eq. (2-5)

The current I is found to be.

$$I = \frac{V}{3.15} + 1.25 \text{ (mA)} \tag{2--6}$$

From the results, we see that 3.15 kQ positive resistance region in this case. As the condition of the change from region 1 to region 2 occurs when  $V_{c1}$  changes from zero to negative value, the limit of

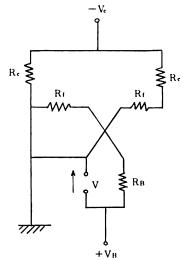


Fig. 2 Equivalent circuit for region 1

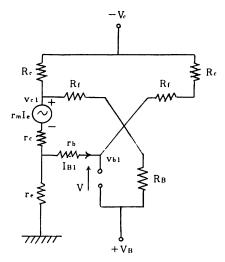


Fig. 3 Equivalent circuit for region 2

### Experiments and Analysis of Negative Resistance

these region takes place at  $v_{c1}=0$ .

By Eq. (2-1)

$$\left(\frac{1}{R_c} + \frac{1}{R_B + R_c}\right) v_{c1} = -\frac{\beta}{r} V + \left(\frac{1}{R_B + R_f} - \frac{\beta}{r}\right) V_B - \frac{V_c}{R_c} = 0$$

voltage V is then

$$V = \frac{r}{\beta} \left\{ \left( \frac{1}{R_B + R_f} - \frac{\beta}{r} \right) V_B - \frac{V_c}{R_c} \right\}$$
 (2-7)

By making substitution of the actual value, V=-3.07V, the current is then I=0.276 mA. Now we can find out the voltage V which is the transfer voltage by setting  $v_{b2}=0$ .

$$v_{b2} = V_B - \frac{V_B - v_{c1}}{R_b + R_f} R_B = 0$$
 (2-8)

From the above Eq.

$$V = \frac{r}{\beta} \left( \left\{ \frac{R_f}{R_B} \left( \frac{1}{R_c} + \frac{1}{R_B + R_f} \right) + \frac{1}{R_B + R_f} - \frac{\beta}{r} \right\} V_B - \frac{V_c}{R_c} \right)$$
(2-9)

With the actual value, V = -2.99V, and the current is calculated by Eq. (2-6), and therefore I = 0.3mA.

(3) Region 3 ··· The nodal Eqs. which result the circuit of Fig.4 are

$$-\left(\frac{1}{R_{B}} + \frac{1}{r} + \frac{1}{R_{f}}\right) v_{b2} + \frac{1}{R_{f}} v_{c1} = -\frac{V_{B}}{R_{B}}$$

$$(3-1)$$

$$-\frac{1}{R} v_{b2} + \left(\frac{1}{R_{c}} + \frac{1}{R_{f}}\right) v_{c1} = -\frac{\beta}{r} v_{b1} - \frac{V_{c}}{R_{c}}$$

$$(3-2)$$

The base voltage  $v_{b2}$  can be calculated by the above two simultaneous equations, i.e.,

$$v_{b2} = -\frac{\frac{1}{R_f} \left( \frac{\beta}{r} v_{b1} + \frac{V_c}{R_c} \right) - \frac{V_B}{R_o R_B}}{\frac{1}{R_o R_k} - \frac{1}{R_{f2}}}$$
(3-3)

Where 
$$\frac{1}{R_o} = \frac{1}{R_c} + \frac{1}{R_f}$$

and 
$$\frac{1}{R_k} = \frac{1}{R_B} + \frac{1}{R_f} + \frac{1}{r}$$

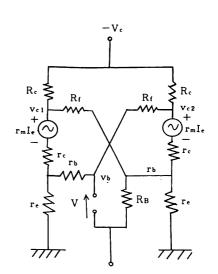


Fig. 4 Equivalent circuit for region 3

Since  $v_{b1}=V+V_B$ , we can see that  $v_{b2}$  falls when V steps up and vice versa. We find out the input current I using the preceding equations:

$$I = \left\{ \frac{1}{R_{f}} + \frac{1}{r} - \frac{R_{o}}{R_{f}^{2}} - \frac{\frac{\beta^{2}R_{o}}{R_{f}^{2}r^{2}}}{\frac{1}{R_{o}R_{k}} - \frac{1}{R_{f}^{2}}} \right\}_{V_{b1}} - \left\{ \frac{\frac{\beta R_{o}V_{c}}{R_{f}^{2}R_{c}r} - \frac{\beta V_{B}}{rR_{B}R_{f}}}{\frac{1}{R_{o}R_{k}} - \frac{1}{R_{f}^{2}}} - \frac{R_{o}}{R_{c}R_{f}} V_{c} \right\}$$
(3-4)

Using Eq. (3-4), then, I can be obtained from the actual values,

$$I = -\frac{V}{0.212} - 13.79 \text{ (mA)} \tag{3-5}$$

We can see by the above equation that the negative resistance comes out at the input terminals in this case, which is  $-212 \Omega$ . Next, the boundary voltage between the region 3 and 4 is given by the condition of  $v_{c2} = 0$ , the value of V is

$$V = -\frac{\frac{\beta R_{o} V_{c}}{R_{f} R_{c}} - \frac{\beta V_{B}}{R_{B}} - \frac{R_{o}}{R_{c}} \left(\frac{r}{R_{o} R_{k}} - \frac{r}{R_{f}^{2}}\right) V_{c}}{\frac{R_{o}}{R_{f}} \left(\frac{r}{R_{o} R_{k}} - \frac{r}{R_{f}^{2}}\right) + \frac{\beta^{2} R_{o}}{R_{f} r}} - V_{B}}$$
(3-6)

The actual values can be calculated by using Eqs. (3-5), and (3-6). Thus

$$V = -2.92 (V)$$
  
 $I = -0.02 (mA)$ 

(4) Region 4 ··· To find the input current I we only use Fig. 5. The current I is then

$$I = \left(\frac{1}{R_f} + \frac{1}{r}\right) V + \left(\frac{1}{R_f} + \frac{1}{r}\right) V_B$$

$$(4-1)$$

Taking the actual values as was used previously, I is

$$I = \frac{V}{3.105} + 0.902 \tag{4--2}$$

and equation tells us that the region is in the positive resistance zone. As the change point from region 4 to 5 is  $V=-V_B$ , if we substitute V=-2.8V into Eq. (4-2), we find I=0.

(5) Region 5 ··· The current I of the circuit of Fig. 6 is found to be

$$I = \frac{1}{R_f} (V + V_B)$$
 (5-1)

From the actual value

$$I = \frac{V}{18.9} + 0.202 \tag{5-2}$$

and we see that the region has the positive value.

#### 3. Experimental Results and Discussion

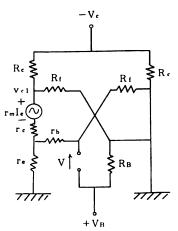


Fig. 5 Equivalent circuit for region 4

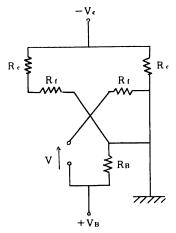


Fig. 6 Equivalent circuit for region 5

In order to find negative input resistance of switching circuit, we measured the volt-ampere characteristics given at the input terminals of the circuit. The experimental result is shown in Fig. 7. In this case, it is found in the experimental results that some errors take place in a certain regions, although the theoretical values and experimental values have a good approximation.

The two most basic sources of experimental errors in the measurements arise because both input resistance  $h_{i\,e}$  and forward current gain  $h_{f\,e}$  are not constant but widely variable parameters by collector voltage and emitter current.

In the process of the analysis of our switching circuit, we regard both of them as the constant parameters, namely,  $h_{ie}=4.0k\Omega$  and  $h_{fe}=80$ , respectively.

Seeking for the relation between hie or hie and emitter current experimentally, we see the follow-

# Experiments and Analysis of Negative Resistance

ing matters; that is, first  $h_{i\bullet}$  changes from  $5.75k\varOmega$  to  $1.77\,k\varOmega$  when the emitter current varies between 0.5 mA and 5.0 mA. Then, the variation of  $h_{f\bullet}$ , as emitter current varies from 0.1 mA to 2.0 mA, is almost from 38 to 82.

In the analysis of some h parameters, as mentioned above, we should not deal with them as constant or, rather they need to be variable. As such, we can understand that experimental errors mainly come out in the ranges of the active states.

# Acknowledgement

The author to thank greatly Mr. S. Ohshima for his valuable comments and suggestions.

## References

(1) L. Strauss. "Wave Generation and Shaping" McGraw-Hill Company, Inc (1960)

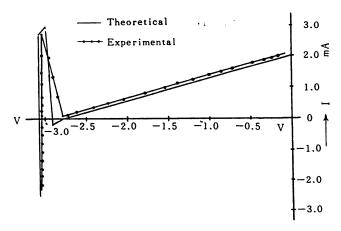


Fig. 7 The negative resistance characteristic given at the input terminals of a switching circuit