An Analysis of RC Oscillator

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1 Introduction

The purpose of this paper is to present an analysis of RC oscillator. Consider the circuit of Fig. 1, where we shall assume that μ amplifier is adjusted so that it will just sustain the oscillations. RC network is characterized by

$$\beta(S)=\frac{e_i}{e_o}=\frac{\dfrac{R/SC}{2/SC+R}}{R+\dfrac{1/SC(1/SC+R)}{2/SC+R}}$$

$$=\frac{\dfrac{R}{SC}}{\dfrac{3R}{SC}+R^2+\dfrac{1}{(SC)^2}}$$

$$=\frac{SCR}{(SCR)^2+3\,SCR+1} \qquad (1)$$

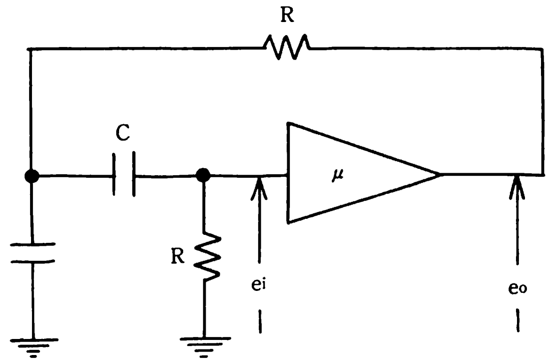

Fig.1 RC orcillator

In the circuit, R is connected on the input side of μ amplifier is values so chosen that the equivalent of $R' \parallel h_i$, R' is the resistor which is actually connected between input and earth side. Substituting $j\omega$ for s results in

$$\beta(\omega)=\frac{1}{3+j(\omega/\omega_0-\omega_0/\omega)} \qquad (2)$$

where $\omega_0=\dfrac{1}{CR}$

The imaginary portion of the denominator of Eq. (2) vanishes at

$$\omega=\omega_0=\frac{1}{CR}\ .$$

At this frequency $\beta=1/3$, and from

$\mu\beta=1$ we find that the gain necessary to sustain oscillation is $\mu=3$.

Next, the basic incremental defining equation may finally be written as a quasi-linear differential equation:

$$\ddot{\nu}+\frac{3-\mu}{CR}\dot{\nu}+\frac{1}{(CR)^2}\nu=0 \qquad (3)$$

This is in the form of the Van der Pol nonlinear differential equation. To find the amplitude of oscillation, the nonlinear differential equation must be solved. We shall find the condition to sustain almost sinusoidal oscillations from the equation. Eq. (3) has two roots, which are located at

$$S_{1,2} = -\frac{3-\mu}{2CR} \pm j\sqrt{\frac{1}{(CR)^2} - \frac{1}{4C^2R^2}(3-\mu)^2}$$

$$= \alpha \pm j\omega \qquad (4)$$

These roots determine the nature of the time-varying response of the oscillator, and therefore we shall investigate their location as a function of μ. In order to have a growing transient, $\alpha > 0$, which corresponds to

$$3 < \mu \qquad (5)$$

2 Analysis of the Systems

We shall determine the nature of the transient response, and examine the location of the root of $1-\mu\beta$. The roots of the circuit of Fig. 1 are found from $1-\mu\beta = 0$. The response will be given by

$$S^2 + \frac{3-\mu}{CR} S + \frac{1}{(CR)^2} = 0 \qquad (6)$$

Eq. (6) has two roots which traverse the path shown in Fig. 2. Even with a system gain margin over the threshold value, the location of the roots will be such that the oscillator will generate a signal of slightly different frequency than that calculated on the basis of the boderline behavior.

If the amplifier gain ever becomes large enough to drive the roots onto the positive real axis, that is, $\mu > 5$, the response will no longer be oscillatory.

Next, Eq. (5) has the restriction which must be imposed on the roots, if the response is to be oscillatory, is that ω must be real. From Eq. (6),

$$4 > (3-\mu)^2 \qquad (7)$$

Fig. 3 presents the relation between $\omega_0^2 [4 - 3(3-\mu)^2]$ and μ. For μ between the limits $1 < \mu < 5$, the transient response will be given by

$$e(t) = A\varepsilon^{\alpha t}(\sin \omega t + \varphi)$$

For small value of α, the natural frequency is almost ω_0.

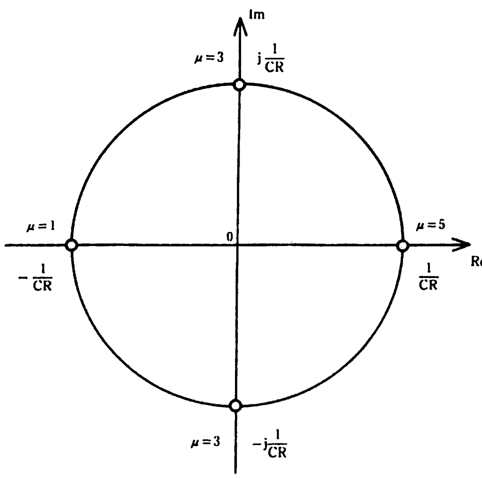

Fig.2 Migration diagram as a function of μ

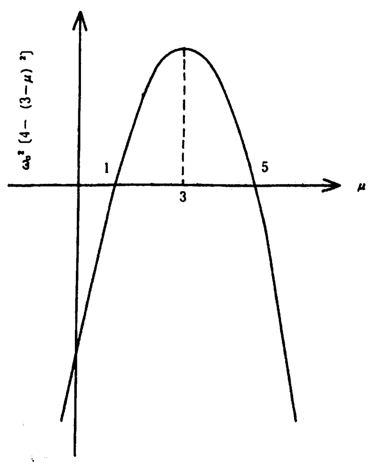

Fig.3 Oscillatory respouse

If μ is less than 3, then any oscillatory response will damp out, which is also called stable center on the phase plane. Once the μ moves from 3 to 5, oscillations occcur and the amplitude of oscillation increases theoretically without limit, but when $5<\mu$, the exponential build-up will lead to a relaxation phenomenon similar to that found for the astable multivibrator.
Then the system response has the unstable nodal point, the amplitude of oscillation forces an increasing aperiodic build up.

3 Phase shift, and Frequency Response

As a measure of frequency stability, we use a quality factor, Q is defined as

$$Q=\frac{\omega_0}{2}\left|\frac{d\theta}{d\omega}\right|_{\omega\to\omega_0} \tag{8}$$

The larger the value of Q, the more stable the oscillator with respect to all changes in the circuit which may produce an undesirable frequency shift. Fig. 4 is a plot of the relations Q and phase behavior of null networks. For this example, the phase of β,

$$\theta=\tan^{-1}\frac{1}{3}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right) \tag{9}$$

Differentiating Eq. (9) with respect to ω and multiplying by $\omega_0/2$ yields

$$Q=\frac{\frac{1}{6}\left[1+\left(\frac{\omega}{\omega_0}\right)^2\right]}{1+\frac{1}{9}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2} \tag{10}$$

At the resonant frequency of the system ($\omega_0=1/CR$), Q reduces to

$$Q=\frac{1}{3} \tag{11}$$

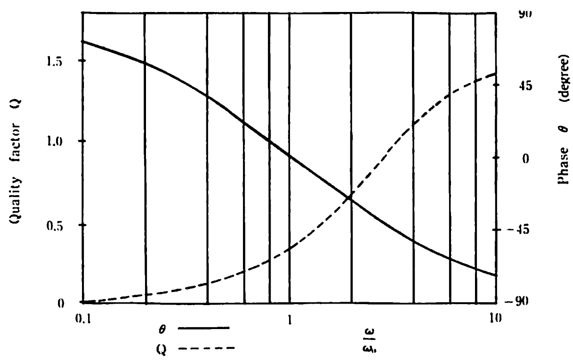

Fig.4 Quality factor and phase characteristics of the circuit of Fig.1

For practical coils with small R_s, the series-Q is

$$Q_s=\frac{\omega_0 L}{R_s}>10$$

With the condition that $Q_s>10$, the equivalent circuit may be found by simplifying the complex impedance equations. The results are that L and C remain unchanged, but the parallel-Q_p is

$$Q_p=\omega CR_p$$

As noted that the curve of amplitude or impedance versus frequency for a series or a parallel resonant circuit, the amplitude or impedance is maximum at resonance, but offers a lower values to the frequency bands on either side. When resonance occurs, a quality factor, Q, of the circuit of Fig. 1 is equal to 1/3. The greater ω, the higher the quality factor Q becomes, finally Q will reach its maximum value at $\omega\to\infty$. Evaluating Q at the limit

$$Q_\infty=1.5 \tag{12}$$

It should be noted that RC oscillator can not reach the maximum Q at resonance, as compared with the LC oscillatory circuit has the maximum Q at the same condition.

4 Conclusion

In this paper, we have dealt with RC oscillator. As seen, Q is only 1/3 in the RC oscillator shown in Fig. 1 at resonance, in spite of the fact that the LC oscillator in general has a higher Q when resonance occurs. Then, it is of importance to note that the maximum Q of the RC oscillator does not express the resonance of it.

References

〔1〕 M.P. Ristenbatt and R.L. Riddle "Transistor Physics and Circuits" Prentice-Hall Inc. 1966.
〔2〕 L. Strauss "Wave Generation and Shaping" McGraw-Hill Book Company, Inc. 1960.

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