Accuracy of Series Solution for Heaving Thin Aerofoil in Incompressible Inviscid Flow

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1. Introduction

Fourier series solution available for various real complex unsteady flow problems is compared with analytical solution [1, 2] which is Söhngen's inversion formula of singular integral equation. The Fourier series for the unknown function is containing the leading edge singularity and satisfying Kutta's hypothesis. Söhngen has found a inversion formula that is very useful and convenient in unsteady flow problems and is derived from potential theory.

In this note the relative error of method of series solution is given for many reduced frequencies and for the various number of control point.

2. Analytical Solution

With the vortex distributions on the wing and wake $\bar{\gamma}_a e^{i\omega t}$ and $\bar{\gamma}_w e^{i\omega t}$, induced velocity $\overline{W}_a(x)e^{i\omega t}$ becomes

$$\overline{W}_a(x) = -\frac{1}{2\pi}\oint_{-b}^{b}\frac{\bar{\gamma}_a(\xi)}{x-\xi}d\xi - \frac{1}{2\pi}\int_{b}^{\infty}\frac{\bar{\gamma}_w(\xi)}{x-\xi}d\xi. \tag{1}$$

After the assumption of simple harmonic heaving motion and the considerable calculation we get the ratio of the lift $\bar{L}e^{i\omega t}$ to quasi-steady lift $\bar{L}_0 e^{i\omega t}$

$$\frac{\bar{L}}{\bar{L}_0} = \frac{1}{2}ik + C(k), \tag{2}$$

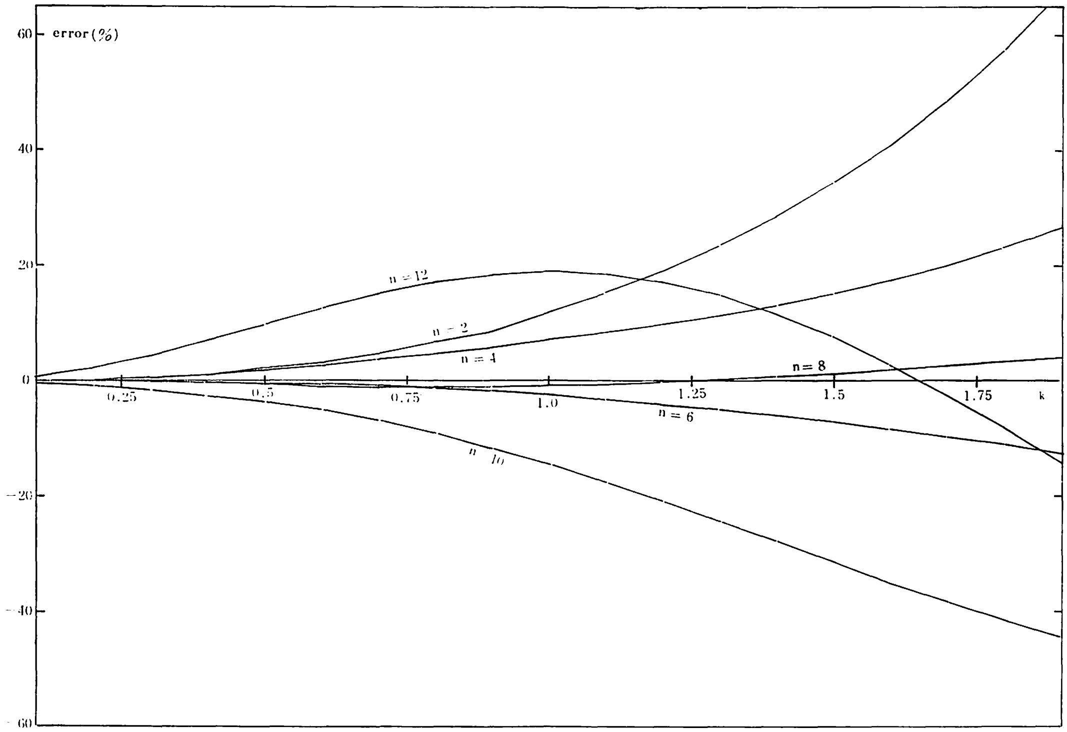
where $$k = \frac{\omega b}{U} \tag{3}$$

is the reduced frequency of oscillation and C(k) is Theodorsen's function.

$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + iH_0^{(2)}(k)} \tag{4}$$

$H_n^{(2)}$ is a combination of Bessel functions of the first and second kinds;

$$H_n^{(2)} = J_n - iY_n \tag{5}$$



Fig,1 Abs〔CL〕

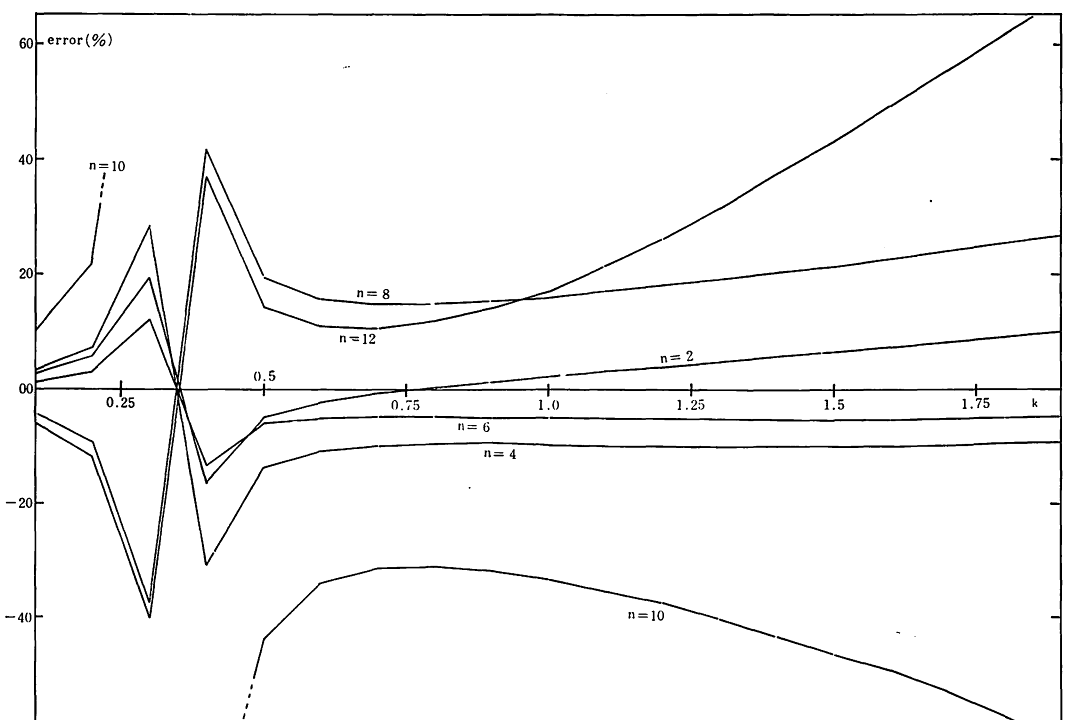


Fig.2 Arg〔CL〕

3. Series Solution

The simple harmonic pressure jump

$$P_U - P_L = \Delta\overline{P}_a(x)e^{i\omega t} \tag{6}$$

induces the perturbed velocity

$$\overline{W}_a(x) = -\frac{\omega}{\rho U^2}\oint_{-b}^{b} \Delta\overline{P}_a(\xi)\, K\left(\frac{k(x-\xi)}{b}\right)d\xi, \tag{7}$$

where

$$K\left(\frac{k(x-\xi)}{b}\right) = \frac{1}{2\pi}\left\{ie^{-i\frac{k(x-\xi)}{b}}\left[Ci\left(\frac{k(x-\xi)}{b}\right) + i\left(\frac{\pi}{2} + Si\left(\frac{k(x-\xi)}{b}\right)\right)\right] - \frac{b}{k(x-\xi)}\right\}. \tag{8}$$

Here Ci and Si represent the cosine and sine integral functions, tabulated by Jahnke and Emde [3] .

In order to solve this Possio's integral equation, unknown function $\Delta\overline{P}_a$ is expanded as follows ;

$$\Delta\overline{P}_a(\theta) = A_0 \cot\frac{\theta}{2} + \sum_{n=1}^{\infty} A_n \sin(n\theta), \quad \frac{\xi}{b} = \cos\theta \tag{9}$$

A fixed number of coefficients should be computed approximately by satisfying Eq. (7) at an equal number of control points on the chord line. Gaussin integration is made of each term.

4. Conclusion

Fig. 1 and 2 represent the relative error of absolute value and phase angle respectively of lift ratio.It is worth notice that the increasing of control points does not ensure the inprovement of accuracy.

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5. References

[1] Bisplinghoff, R. L., Ashley, H. and Halfman, R. L, Aeroelasticity, Addison-Wesley, 1957

[2] Fung, Y. C., Theory of Aeroelasticity, Dover, 1968

[3] Jahnke, E. and Emde, F., Tables of Functions with Formulas and Curves, Dover, 1945