Influence of Joints on Rock Mass Behavior

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1. Preface

Major problems in rock mechanics can't be avoided, especially the situations of discontinuities and anisotropies of rock masses. For these problems, we have several questions, most of which have already been analyzed by former researchers. A partial description is as follows:

1) will whether failure take place along the weakness plane under the applied stresses ?
2) under which stress conditions will failure take place along the weakness planes ?

These are the most common problems, and now yet it is important to judge dangerous direction of weakness planes with respect to a stress system. To a joint system represented by Coulomb-Navier's criterion, the coefficient of joint is applied, and in the final section concerning the laboratory test, several possible results are represented.

2. Theories of Stress Conditions in a Weakness Plane

It is important to find the condition of failure which may occur in the weakness planes such as seams, joints and faults. In such weakness planes, if the shear strength is subject to Coulomb-Navier's criterion, the following equation often used;

$$S_0 = C_0 + \sigma \tan\varphi_0 \qquad (2.1)$$

where, S_0 : the shear strength of the solid (rock),
C_0 : the cohesion of the rock,
φ : the angle of internal friction for rock,
σ : the normal stress across the plane.

If τ is the shear stress, the condition for failure is given by

$$|\tau| \geqq C_0 + \sigma \tan\varphi_0 \qquad (2.2)$$

Accordance with Jaeger's consideration[(1)] with respect to a system of weakness plane in a two dimension like Fig. 1, let C = cohesion of joint filling materials, φ = angle of internal friction, we get instead of eq. (2.2)

$$|\tau| \geqq C + \sigma \tan\varphi \qquad (2.3)$$

Let's average normal stress σ_{av} and maximum shear stress τ_{max},

$$\sigma_{av} = \frac{\sigma_1 + \sigma_3}{2}, \qquad \tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \qquad (2.4)$$

in which, principal stresses $\sigma_1 > \sigma_3$ (positive shows compression),
consequently, the shear stress and normal stress at the weakness plane which is inclined β to the maximum principal stress, as shown in Fig. 1,

$$\left.\begin{aligned} |\tau| &= \tau_{max} \sin 2\alpha \\ \sigma &= \sigma_{av} + \tau_{max} \cos 2\alpha \end{aligned}\right\} \quad (2.5)$$

homogeneous rock takes place failure when $f(\alpha) = \tau - \sigma \tan\varphi$ approaches to a maximum value, from $\partial f(\alpha) / \partial\alpha = 0$

$$\alpha = 45° - \frac{\varphi}{2} \quad (2.6)$$

Substituting eq. (2. 5), (2. 6) to (2. 3), the condition for failure along the weakness plane can be gotien as:

$$\begin{aligned} \tau_{max} &\geqq (\sigma_{av} + C \cdot \cot\varphi) \sin\varphi / \sin(2\alpha+\varphi) \\ &= (\sigma_{av} + C \cdot \cot\varphi) \tan\theta \end{aligned} \quad (2.7)$$

in which, $\tan\theta = \dfrac{\sin\varphi}{\sin(2\alpha + \varphi)}$

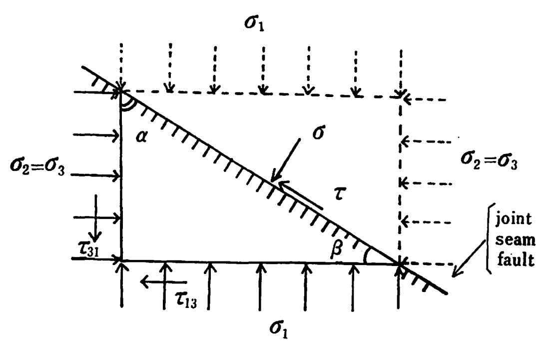


Fig. 1 Stress diagram in a weakness plane

and eq. (2.7) can be rewritten as follows:

$$\begin{aligned} \sigma_1 [\sin(2\alpha+\varphi) - \sin\varphi] - \sigma_3 [\sin(2\alpha+\varphi) \\ + \sin\varphi] \geqq 2C \cdot \cos\varphi \end{aligned} \quad (2.8)$$

or $$\sigma_1 \cos(\alpha+\varphi) \sin\alpha - \sigma_3 \sin(\alpha+\varphi) \cos\alpha \geqq C \cdot \cot\varphi \quad (2.9)$$

Talobre J. gives next eqaution[3] in the case of $\beta = \dfrac{\pi}{2} - \alpha$

$$\sigma_1 \sin(\varphi-\beta) \cos\beta + \sigma_3 \cos(\varphi-\beta) \sin\beta + C \cdot \cot\varphi \leqq 0$$

In the case of smooth horizontal surface, from eq. (2. 5),

$$\cos 2\alpha = \sqrt{1-\sin^2 2\alpha} = \sqrt{1 - \frac{\tau^2}{\tau^2_{max}}}$$

Substitution of this in the 2nd of eq. (2. 5) gives

$$\sigma - \sigma_{av} = \tau_{max} \sqrt{1 - \frac{\tau^2}{\tau^2_{max}}}$$

$$\tau^2_{max} = (\sigma - \sigma_{av})^2 + \tau^2 \quad (2.10)$$

Eq. (2.10) means the equation of circle[4] having a radius $\tau_{max} = \dfrac{1}{2} (\sigma_1 - \sigma_3)$

and centered on the σ – axis at $\sigma_{av} = \dfrac{1}{2} (\sigma_1 + \sigma_3)$

When $\varphi = 0°$, giving $\sigma = \sigma_1$ & $\tau = 0$.

In the above equations, if $C = 0$, then mechanically,

$$|\tau| \geqq \sigma \tan\varphi$$

The condition of failure in the weakness plane will be :

$$\left.\begin{aligned} &\tau_{max} \geqq \sigma_{av}\frac{\sin\varphi}{\sin(2\alpha+\varphi)} \\ \text{or}\quad &\sigma_1[\sin(2\alpha+\varphi)-\sin\varphi]-\sigma_3[\sin(2\alpha+\varphi)+\sin\varphi]\geqq 0 \end{aligned}\right\} \quad (2.11)$$

If $\varphi = 0$, then the condition for failure depends upon cohesion, $|\tau| \geqq C$, similarly,

$$\left.\begin{aligned} &(\sigma_1-\sigma_3) \geqq \frac{2\cdot C}{\sin 2\alpha} \\ \text{or}\quad &\tau_{max} \geqq \frac{C}{\sin 2\alpha} \end{aligned}\right\} \quad (2.12)$$

Jaeger (1960)[2] has considered the effect of a more general type of anisotropy upon shear failure. In the more general case, he assumed that the cohesive strength of the material is equal to the cohesive strength of the weakness plane and the least value lies on the weakness plane. On the other hand, a maximum value rotates through a further 90°. Thus when the cohesive strength in the weaknwss plane inclines β to the maximum principal stress it then becomes :

$$C = C_1 - C_2\cos 2(\omega-\beta) \quad (2.13)$$

Hence, when the cohesive strength in the plane which is inclined ω to the maximum stress has a minimum value $C_1 - C_2$ when $\omega = \beta$, and a maximum value of $C_1 + C_2$ then the plane of anisotropy is rotated through a further 90°. Using this expression for the cohesive strength, the condition of failure becomes :

$$|\tau| = C_1 - C_2\cos 2(\omega-\beta) + \sigma\tan\varphi \quad (2.14)$$

We can rewrite this by using eq. (2. 5)

$$\begin{aligned} &\tau_{max}\sin(2\beta+\varphi) + C_2\cos 2(\omega-\beta)\cos\varphi \\ &\quad = C_1\cos\varphi + \sigma_{av.}\sin\varphi \end{aligned} \quad (2.15)$$

Since eqs. (2. 14) and (2. 15) are the same, the conditions of failure are thus similarly described previously :

$$\tan 2\beta = \frac{\tau_{max} + C_2\sin 2\omega}{\tau_{max}\tan\varphi + C_2\cos 2\omega} \quad (2.16)$$

3. A Consideration to Joint System

In the former section, conditions of failure are introduced for weakness planes. Now joint planes including the coefficient of joint shall be considered here. In this case, we can use eq. (2. 3)

$$|\tau| \geqq C + \sigma\tan\varphi$$

The same type of relationship may be applied to a plane of weakness. In this case, the joint would develop resistance against shear force in the term of :

$$\tau_j = \sigma\tan\varphi_j \quad (3.\ 1)$$

in which, τ_j ; joint friction

φ_j ; angle of joint friction

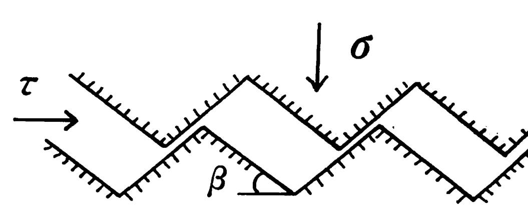


Fig. 2 Model Joint Surface

In the case of Fig. 2, roughness of joint influences the shear resistance under the condition $\varphi_j \ll \beta$, then the eq. (3. 1) becomes :

$$\tau_j = \sigma \tan(\beta + \varphi_j) \qquad (3.2)$$

Thus the strength influenced by joint can be drawn in Fig. 3. As shown in Fig. 2, most of the joint is not continuous, thus it is assumed that the coefficient of joint "k" which is the ratio of area of open joint and total area.

"k" usually takes 0.3 to 0.7, then the eq. (2.19) becomes :

$$\tau_j = k \cdot \sigma \cdot \tan(\beta + \varphi_j) \qquad (3.3)$$

Owing to this joint friction, total shear strength will be much higher. Therefore, it may be written in the form of

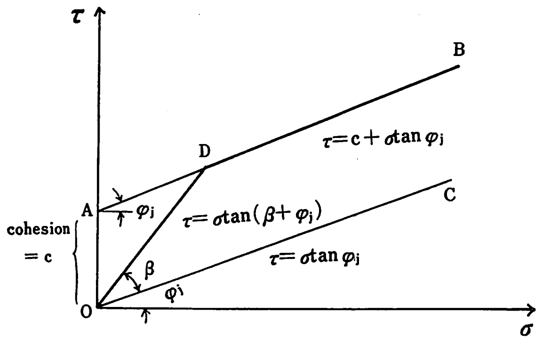


Fig. 3 Jointing mass strength

OD : shear occurs along the joint, having inclination β to the shear force direction

OC : the force coincides with the joint direction (β=0)

AB : the displacement occurs as a result of the monolitic block shearing.

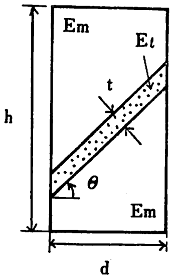
$$S_0 = C_0 + (1-k)\sigma \cdot \tan\varphi_0 + k \cdot \sigma \cdot \tan(\beta + \varphi_j) \qquad (3.5)$$

If the joint plane is wet, pore water pressure (u) would take place, thus affecting the total strength is as follows :

$$S_0 = C_0 + (1-k)(\sigma - u)\tan\varphi_0 + k(\sigma - u)\tan(\beta + \varphi_j) \qquad (3.5)$$

4. An Example of a Laboratory Test[1], [2]

As a method of expression of weakness or discontinuities of rocky material in the laboratory test, an experimental study is carried out to investigate the uniaxial compressive strength and the mechanism of fracture of cylindrical specimens (Fig.4) with an inclined layer made by material of lower strength. A part of the results obtained from the laboratory test is shown below.



θ=0°, 15°, 30°, 45°, 60°

h=10cm

d= 5 cm

E_m/E_l= 3 (Young's Modulus ratio)

t : thickness of the layer (cm)

Fig. 4 Laboratory test specimen

[Materials]

Main part : plaster + H_2O (1 : 0.6 in weight ratio)

Layer part : plaster+diatomaceous earth+H_2O (1 : 0.1: 1 in weight ratio)

4—1. Mechanism of Fracture

Through the experiments, it was observed that the mechanism of fracture was divided into three cases as follows :

1) θ = 0°, 15°, 30°

In this case, the first slip fracture appears in the layer part and the fracture of the main part arises when the slip line of the layer part reaches its boundary.

2) θ = 45°

The slip fracture of the layer part yields to the fracture of the total body.

3) θ = 60°

The slip fracture appears at the boundary of two parts. Skeches of mechanism of fracture are shown in Fig. 5.

4—2. Theories

As a result of additional triaxial test, Mohr's failure envelope reached to $\tau = \tau_0$ (const), i.e., the following assumptions may be based on the theory of maximum shear stress.

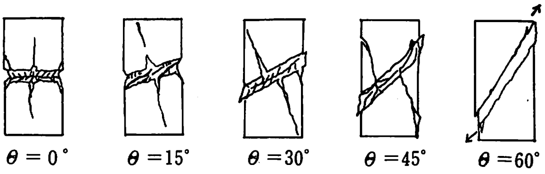


Fig. 5 Skeches of specimens at failure

Ⓐ If it is assumed that the failure occurs when $\tau = \tau_0$ at the angle of inclined layer (Fig. 6), and also assume the axial stress is σ_1, the shear stress acting on a plane inclined θ is thus :

$$\tau = \frac{1}{2} \sigma_1 \sin 2\theta \tag{4. 1}$$

$$\tau_0 = \frac{1}{2} \sigma_D \tag{4. 2}$$

then,
$$\sigma_1 = \sigma_D / \sin 2\theta \tag{4. 3}$$

Ⓑ Assume the restriction force between top and bottom of the parting layer is to take place (Fig. 6). Besides σ_1, it is assumed that the principal stress $\sigma_2 = \sigma_3$ arises to the layer part thus :

$$\varepsilon_1 = \frac{1}{E_D} (\sigma_1 - 2\nu_D \sigma_2) \tag{4. 4}$$

$$\varepsilon_2 = \frac{1}{E_D} (\sigma_2 - \nu_D \sigma_2 - \nu_D \sigma_1) \tag{4. 7}$$

in which, ε_1, ε_2 : strain with respect to σ_1, σ_2 in the layer part.

E_D, ν_D : Young's Modulus and Poisson's ratio of the layer part respectively.

ε_2 is assumed equal to ε_2' which is the strain of main part in the vicinity of contact plane. We can measure ε_1, ε_2' at the laboratory, so the relation of principal stress is thus calculated to be :

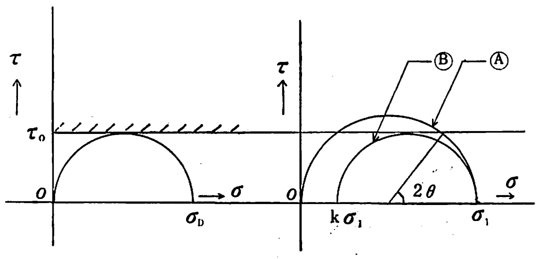


Fig. 6 Mohr's Circles

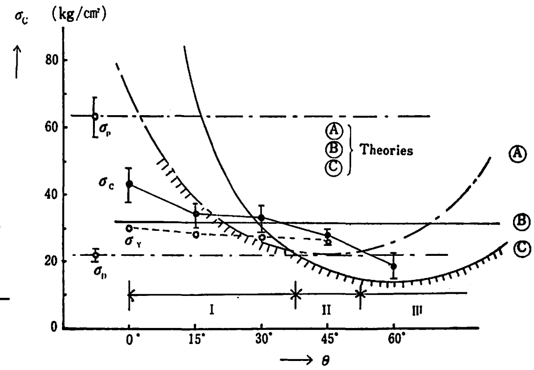


Fig. 7 Theory and Experimental results

$$\sigma_2 = k \cdot \sigma_1 \quad \left(0 < k = \frac{\alpha + \nu_D}{1 - \nu_D + 2\nu_D} < 1, \qquad \alpha = \frac{\varepsilon_2'}{\varepsilon_1}\right) \tag{4. 6}$$

$$\sigma_1 = \sigma_Y = \frac{\sigma_D}{1 - k} > \sigma_D \tag{4. 7}$$

Ⓒ Assume large cracks lie on the contact plane of two parts, based on Griffith's theory. If tensile stress (σ_t) acting on cracks reaches uniaxial tensile strength of the contact plane, so under uniaxial stress state,

$$\sigma_1 = \frac{-4\sigma_t}{2\sqrt{\frac{1}{2}(1+\cos 2\theta)-(1+\cos 2\theta)}} = \frac{-2\sigma_t}{\cos(1-\cos\theta)} \qquad (4.8)$$

then, $\theta = 60° \longrightarrow \sigma_1 = -8\sigma_t$, $\theta = 45° \longrightarrow \sigma_1 = -9.74\sigma_t$.

Fig. 7 shows above results.

Description in Fig. 7 :

σ_C : compressive trength

σ_Y : stress at yield point

σ_P : compressive strength of main part material

σ_D : compressive strength of layer part matarial

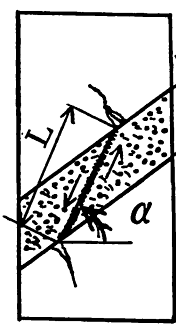


Fig. 8 Failure State

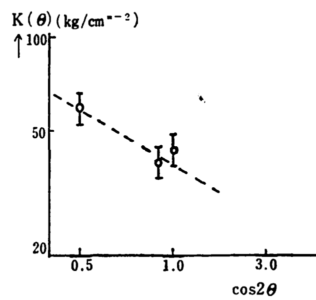


Fig. 9 Variation of Function of angle

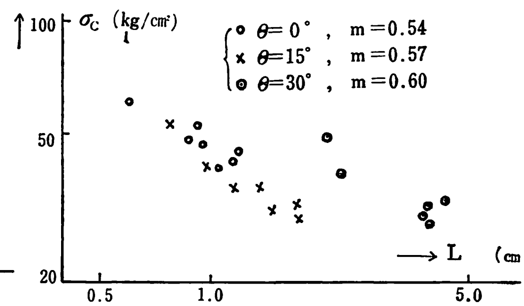


Fig. 10 σ_C–L Relations

In which, I, II and III show the failure tendencies of model specimens.

4—3. The case of $\theta = 0°$ to $30°$

As shown in Fig. 8, the specimen with $\theta = 0°$ to $30°$ almost shows $\alpha = 45°$, so σ_C is closely related with L.

Ⓐ Relation of L and σ_C

$$L = \frac{\Delta t}{\sin\left(\frac{\pi}{4} - \theta\right)} \qquad (\Delta t \text{ is t at failure}) \qquad (4.9)$$

Fig. 9 shows the relation σ_C and L, and from this figure, experimental equation can be deduced as :

$$\left.\begin{array}{l}\sigma_C = K(\theta)\, L^{-m} \\ (\text{m average} = 0.57)\end{array}\right\} \qquad (4.10)$$

in which K (θ) is a function of angle and it varies with inclined angle.

Ⓑ Relation of θ and σ_C

For solving the function of K (θ), relation $\cos 2\theta$ and K (θ) = $\sigma_C\, L^m$ drawn in Fig. 10, which gives :

$$\left.\begin{array}{l}K(\theta) = K_0 (\cos 2\theta)^{-n} \\ (K_0 = 40.4,\ n = 0.58)\end{array}\right\} \qquad (4.11)$$

These results are summerized in the following eqaution :

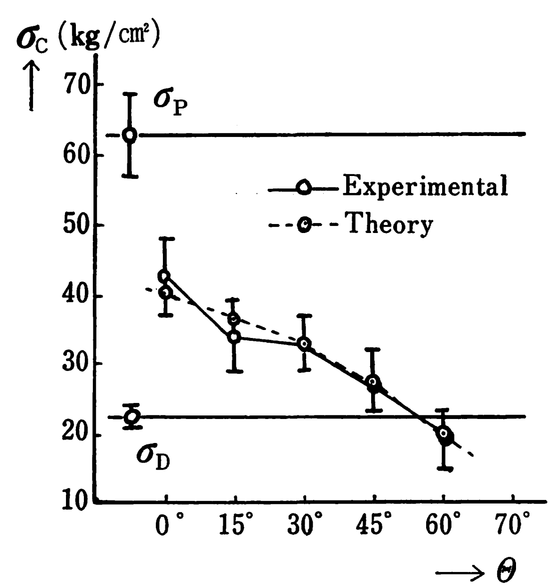


Fig. 11 Conpressive Strength

$$\sigma_C = \frac{K_0}{L^{0.57}\,(\cos 2\theta)^{0.58}} \tag{4.12}$$

Physically, this becomes :

$$\sigma_C = \frac{K_0}{\sqrt{L \cdot \cos 2\theta}} \tag{4.13}$$

Substitution of already mentioned L in this eqaution, thus :

$$\sigma_C = \frac{K}{\sqrt{4\pi \cdot \cos\left(\frac{\pi}{4} - \theta\right)}}$$

$$(K = K_0/\sqrt{2}) \tag{4.14}$$

This result is shown in Fig. 11.

5. Conclusions

The problem of influence of joints is quite complex. In analizing this theoritically, various kinds of assumptions can be made. Since this is the case, we often have to consider additional phenomena. Rock mass has several different structure types, and generalization is impossible. Therefore, new models must be fabricated in order to facilitate analytical assumptions of the derivative conclusions.

More comparisons will thus be necessary for value analysis in respect to laboratory testing for criterion for failure.

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